Private Label Positioning:

Vertical vs. Horizontal Differentiation from the National Brand

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May 2004

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Abstract

Among major benefits private labels bring to the retailer, we focus on the retailer's ability to coordinate the prices of both the national brand and its store brand counterpart. By using product-line pricing, the retailer can exploit the differentiated nature of the two brands. This paper investigates the retailer's problem of positioning her private label and the subsequent pricing issues. While most previous studies concentrate either on vertical (i.e., quality) or horizontal (i.e., trade dress) differentiation, we view the private label's differentiation from the national brand as a combination of the two. In contrast to the location theory literature, which generally suggests maximum differentiation in both dimensions as the best strategy for the retailer, our results are mixed: when the two dimensions are considered separately, the retailer's profit is maximized by minimum vertical differentiation and maximum horizontal differentiation. However, when the two dimensions are correlated, the retailer is generally better off increasing its quality up to the level of the national brand, provided that the quality cost is negligible. We also extend the model to the case of two national brands, and show that the private label's optimal level of differentiation depends on the existing differentiation between the two brands.

KEY WORDS: Private Label, National Brand – Private Label Competition, Private Label Positioning, Vertical and Horizontal Differentiation.

INTRODUCTION

By 1999, private label products (or equivalently, store brands) accounted for over 20 percent of supermarket unit sales and 15.7 percent of dollar sales (Williams 2000). In 77 of 250 supermarket product categories, private labels in the U.S. collectively have higher unit market shares than their strongest nationally branded competitor, while they are second or third in 100 of those categories (Quelch and Harding 1996). Among other benefits, private labels add diversity to a retailer's product line in a category (Raju, Sethuraman and Dhar 1995; Soberman and Parker 2004). This diversity – or product differentiation – can be *vertical* or *horizontal* in nature (or both, of course). Changes in both dimensions have been partially responsible for private label sales growth in U.S. grocery retailing (Wellman 1997 refers to improvements in packaging and selection as well as quality in the late 1980s as driving forces).

Vertical differentiation is essentially quality-driven, where private labels are usually perceived as a lower quality substitute to the corresponding national brands (e.g., Ann Page canned soups – the A&P grocery chain's former private label – were widely viewed as lower quality than Campbell's canned soups).[1] Vertical differentiation is based on the notion that the characteristic on which differentiation occurs is one for which *all* consumers value the highest possible level. A superior quality national brand may lose its vertical differentiation from the private label if private-label retailers are eventually able to match the national brand's technology and perception (e.g., President's Choice cookies). However, the economics literature indicates that a second product—a private label in our context—in the market would find it optimal to vertically differentiate itself from the market leader (Shaked and Sutton 1982; Moorthy 1988). In particular, using a uniform distribution of consumer's willingness to pay for quality, Moorthy obtains a quality-price equilibrium where the two brands are positively but finitely separated. A key assumption in his analysis is a quadratic cost function: The finite, market-uncovered equilibrium arises because the firms find it too costly to produce a product with a quality higher than a certain level. Desai (2001), on the other hand, assumes a model with two distinct quality segments, in which firms offer different quality products for different markets. Desai uses Hotelling's location model structure to capture quality-price competition in monopoly and duopoly settings.

In contrast, horizontal differentiation means that products may have different forms, sizes, or packaging, but there is not a uniform ideal point for such an attribute. The economics literature is divided between two opposite results when there are two brands of equal quality: minimum differentiation (Hotelling 1928) and maximum differentiation (d'Aspremont, Gabszewicz, and Thisse 1979), depending on the underlying assumptions. Sayman, Hoch, and Raju (2002) add a third brand (a private label) in their model, in which the two incumbent national brands are assumed to be maximally differentiated in the horizontal dimension. They find that it is optimal for the private label to imitate the stronger national brand. Positioning at a midpoint would never be optimal. For example, a private label mouthwash would imitate either Listerine or Scope, but never position somewhere in between the two. In addition, their empirical study reveals that a store brand that imitates a national brand (horizontal differentiation) may not have much impact on quality perceptions (vertical differentiation). Du, Lee, and Staelin (2004) similarly model two national brands and one store brand, but in a location model structure derived from consumer utility. They examine retailer private-label positioning strategies assuming fixed maximum category demand and a constraint on the horizontal position of the private label (specifically, the private label may position only between the two national brands).

Our model investigates a similar context of national brand-private label competition, but we investigate *both* vertical and horizontal differentiation using a utility theory-based demand function that allows for flexible category demand. One of our key findings is that, when the two national brands are *horizontally undifferentiated*, the retailer is better off by maximally differentiating her private label from the national brands. Meanwhile, when they are *horizontally differentiated*, the private label's optimal strategy is to imitate one of the national brands. This indicates that there is no single best differentiation strategy for a private label in all market conditions, which is supported by the real market examples summarized in Table 1.

[Insert Table 1 about here]

Many private label retailers have purposely sought to *minimize* horizontal differentiation from the national brand, by making their packaging, sizes, typeface, and labeling extremely similar to those of the national brand. This strategy gave rise in the mid-1990s to a trademark lawsuit by Unilever, the maker of Vaseline Intensive Care Lotion, against Venture Stores, Inc., alleging that the virtually identical product form and "trade dress" (the legal term for the brand, trademark, logo, and packaging that identify a product) of Venture's lotion constituted trademark infringement (*Conopco, Inc. v. May Department Stores Co.*, 1995; see also Harvey, Rothe, and Lucas 1998 and Harvey, Rothe, Kasulis and Lucas 1999 for discussions in the Marketing literature). The court ruled in Venture's favor, saying that the Venture store brand's label explicitly invited the consumer to compare Venture's product with Vaseline Intensive Care. One of the core conditions for trademark infringement, that consumers would be confused into thinking the imitator is in fact the original branded product, therefore did not hold. Hence, the legality of virtual imitation of the national brand by the private label (i.e., the *minimization* of horizontal differentiation) was upheld.

Two interesting features of this case deserve mention in our context. First, the principle upheld was the legality of *minimum* horizontal differentiation; but one of the basic insights of location modeling is the optimality of *maximum* horizontal differentiation. Second, while the apparent issue is pure horizontal differentiation through trade dress imitation, it would seem intuitively that Venture was also trying to appear to offer a product of similar *quality* to its consumers and to signal this through similar trade dress. This suggests that in many real-world cases, the retailer's positioning of the private label inherently involves both horizontal and vertical positioning messages to its consumers. The concept of differentiation of the private label from the national brand is thus not a unidimensional concept. The overall interchangeability or substitutability between them is a function of both vertical and horizontal product differences, and the retailer's positioning choice for its private label will naturally reflect both dimensions.

The primary focus of this paper is an investigation of the optimal vertical and horizontal positioning strategies of the private-label retailer that also sells the national brand in a given category. We use a standard consumer utility maximization foundation to build our analysis, as it allows for *both* vertical and horizontal differentiation between products. We show that the optimal positioning strategy can be quite different from that suggested by the location modeling literature, suggesting that it is important to consider category by category whether total consumption is essentially fixed, or whether it can be expanded with product variety investments.

A related model was proposed by Vandenbosch and Weinberg (1995), who assume a two-dimensional attribute space. Their equilibrium solution is "max-min" differentiation in which the strong brand locates at the best position and the second brand chooses maximum differentiation in one attribute dimension and minimum differentiation in the other in a large part of the parameter space. Note that their model has two vertical dimensions, whereas ours has one vertical and one horizontal dimension. In addition, theirs is a location model with fixed total category demand, as opposed to our flexible demand utility-based model.

In what follows, we first introduce the base model with one national brand. It is shown that the private label is better off differentiating horizontally from the national brand counterpart. Then we extend the model to the case in which there are two national brands. When the national brands are substantially differentiated and the private label's potential position is limited to the space between the two national brands, our result is the same as Sayman et al.'s (2002): it is better to imitate the stronger national brand. On the other hand, when the two national brands are horizontally undifferentiated, the result is the same as that of our base case: it is better for the private label to horizontally differentiate from the national brands.

THE DEMAND MODEL AND THE RULES OF THE GAME

In this section, we present a duopoly demand function derived from a consumer utility framework. This is a base demand function with one national brand and a private label. In a later section, we will extend this base model to a more general case of two national brands. Our consumer utility function for two substitutes takes the following standard quadratic form widely used in economics (Häckner 2000):

$$U(q_1, q_p) = (\alpha_1 - p_1)q_1 + (\alpha_p - p_p)q_p - (1/2)(\beta_1 q_1^2 + \beta_p q_p^2 + 2\gamma q_1 q_p), \qquad (1)$$

where q_i is the quantity of product i consumed and p_i is the price of product i (i=1, p), and subscript p represents the private label. The other parameters have standard interpretations in utility theory, that can be seen by examining the marginal utility of consumption for product i:

$$\frac{\partial U}{\partial q_i} = \alpha_i - \beta_i q_i - \gamma q_j - p_i, \quad i = 1, p.$$

Clearly, the higher is α_i, the higher is the marginal utility of consumption of product *i*; thus, α_i represents the intrinsic value or quality of product *i*. Without loss of generality, we assume that $\alpha_1 > \alpha_p$. The difference between α_1 and α_p is a measure of the *vertical differentiation* between the two products. The parameter β_i measures the rate at which the marginal utility of consumption for product *i* declines with units of product *i* itself consumed. It is natural to assume that $\beta_1 < \beta_p$, reflecting a steeper marginal utility decline for the less valued private label.

Finally, the parameter $\gamma \in [0, \beta]$ measures the rate of decline of marginal utility of consumption for product *i* with respect to the consumption of the other product, *j*. This parameter thus represents the interchangeability or substitutability of products 1 and p, and is a measure of horizontal differentiation (Häckner 2000). When $\gamma = 0$, the two products are independent; and when $\gamma = \beta$, the products are perfect substitutes. Therefore, the utility function (1) captures both vertical and horizontal product differentiation between the national and store brands. The assumption of decreasing marginal utility allows the consumer to optimally allocate his budget between the two differentiated products.[2]

The utility-maximizing consumer will optimally allocate the quantities consumed by solving the first-order conditions: $\frac{\partial U}{\partial q_1} = 0$ and $\frac{\partial U}{\partial q_p} = 0$. The second-order conditions for a utility maximum require that $\beta_1 \beta_p > \gamma^2$; this condition intuitively implies that cross-product utility effects are not as large as own-product utility effects.[3] The optimal solution results in the following demand system:

$$q_1 = \frac{1}{\beta_1\beta_p - \gamma^2}[(\alpha_1\beta_p - \alpha_p\gamma) - \beta_p p_1 + \gamma\, p_p] \tag{2}$$

$$q_p = \frac{1}{\beta_1\beta_p - \gamma^2}[(\alpha_p\beta_1 - \alpha_1\gamma) - \beta_1 p_p + \gamma\, p_1] \tag{3}$$

In order to guarantee positive demand when all prices are zero, we require that $\alpha_1\beta_p - \alpha_p\gamma > 0$ and $\alpha_p\beta_1 - \alpha_1\gamma > 0$. Note that these demand functions are in the same linear form as that of the most popular linear demand function, but the coefficients of the demand functions are explicitly expressed in terms of the underlying utility parameters. It is easy to see that changing the substitutability parameter γ, for example, nonlinearly affects the intercept, the own-price derivative of demand, and the cross-price demand derivative. Therefore, our demand structure provides clear insights into the effects of changes in utility parameters on market outcomes. Other commonly-used demand functions in the marketing literature cannot provide these insights because they are not derived directly from utility maximization.

While the national brand manufacturer determines the wholesale price (w_1) for his product 1 whose quality level is α_1, the retailer's decision is to choose the optimal level of vertical differentiation (α_p) of her store brand from the national brand, the degree of (horizontal) substitutability (γ) between the national and store brands, the national brand's retail margin (m_1, equal to $p_1 - w_p$), and the retail price (p_p) for the private label. Since the focus of our paper is on the retailer's decisions, we assume that α_1 is already given. Observing the quality level of the national brand and *anticipating* subsequent price equilibrium, the retailer vertically and horizontally positions her private label. Thus, the equilibrium concept is a sub-game perfect equilibrium, in which the second-stage price equilibrium is reached immediately after the differentiation decisions. Price competition is modeled as a manufacturer-Stackelberg game, in which the manufacturer sets the national brand's wholesale price (w_1) anticipating the retailer's pricing reaction that sets the retail margin (m_1) for the national brand and the retail price (p_p) of her private label.

THE SHORT-RUN PRICE EQUILIBRIUM

In this section, we assume the two brands' quality levels α_1 and α_p to be fixed and derive the retailer's optimal pricing strategy and equilibrium values of all prices, quantities, and profits in the system. The retailer is assumed to use product-line pricing as part of category management. The manufacturer-Stackelberg pricing game is solved recursively, starting from the retailer's pricing decision. The retailer maximizes her combined profit:

$$\text{Maximize}_{m_1, p_p}\ \Pi_{\mathrm{R}} = m_1 q_1 + (p_p - v_p) q_p, \qquad (4)$$

where ν_p is the unit variable cost of the private label, which is an increasing function of α_p (that is, creating a higher-quality private label product results in a higher marginal cost of production).[4] Solving the first-order conditions ($\frac{\partial \Pi_R}{\partial m_1} = 0, \frac{\partial \Pi_R}{\partial p_p} = 0$) for m_1 and p_p, and then adding m_1 and w_1 to obtain p_1, we obtain the following retailer reaction functions:

$$p_1 = \frac{\alpha_1 + w_1}{2} \text{ and } p_p = \frac{\alpha_p + \nu_p}{2}. \tag{5}$$

To guarantee that the retail margin is positive, we require that $\alpha_1 \geq w_1$. It is interesting to note that p_p is independent of the national brand's wholesale price, even though the two products are interrelated in demand. The retailer chooses the private label's price based only on its own quality and variable cost.[5] That is, product-line pricing implies that it is in the retailer's best interest not to link the private label's price to the level of the national brand's wholesale price:

Proposition 1. When the retailer employs product-line pricing, the private label's optimal price is independent of the national brand's wholesale price.

The retailer's optimal margin for the national brand ($p_1 - w_1$) is negatively related to its wholesale price and positively related to its intrinsic quality. When the wholesale price increases, the retailer absorbs half the increase and passes through only the other half to the national brand's retail price. This pass-through rate is consistent with McGuire and Staelin's (1983) and Coughlan's (1985) buffer effect and Lee and Staelin's (1997) vertical strategic substitutes property of the linear demand function.

Given the retailer's reaction functions (5), the manufacturer's pricing decision is to choose his wholesale price so as to maximize his short-term profit:

$$\text{Maximize}_{w_1} \quad \Pi_M = (w_1 - \nu_1) q_1(m_1(w_1), p_p) \tag{6}$$

where ν_1 is his unit variable cost ($w_1 \geq \nu_1$). Solving the first order condition, we have the optimal wholesale price:

$$w_1^* = \frac{\beta_p(\alpha_1 + \nu_1) - \gamma(\alpha_p - \nu_p)}{2\beta_p}. \tag{7}$$

It is straightforward to verify the second order condition. The optimal wholesale price increases positively with the national brand's quality (α_1), its unit variable cost (ν_1), and the private label's variable cost (ν_p). It is negatively related to the private label's quality (α_p). All these directional changes make intuitive sense. Note that, to make an optimal wholesale pricing decision, the manufacturer needs to have information on all the utility parameters except β_1. Since we know $\alpha_1\beta_p - \alpha_p\gamma \geq 0$ from the above discussion, w_1 is nonnegative. We can obtain all related equilibrium values by substitution, which are summarized in Table 2.

[Insert Table 2 about here]

DIFFERENTIATION OF THE PRIVATE LABEL

Given the short-run price equilibrium and the quality level of the national brand, the retailer's long-term decision is to choose the position of her private label. The positioning decision in this paper is different from that of Sayman, Hoch, and Raju (2002) or Du, Lee, and Staelin (2004), in which the private label's horizontal position is constrained to lie between that of the two national brands. Our model lets the retailer choose both vertical and horizontal dimensions of differentiation in the context of the examples in Table 1. In what follows, we examine the effects of vertical and horizontal differentiations as well as their combined effects.

Effects of Vertical Differentiation

For the moment, we fix the value of γ, and consider the retailer's problem of choosing the quality (α_p) of her private label, anticipating the price equilibrium in the short run. From Table 2, it is straightforward to show that the equilibrium demand for the private label (q_p) is positive, and is an increasing function of α_p, for α_p greater than $\underline{\alpha}_p$:

$$\underline{\alpha}_p = \frac{\beta_p\gamma(\alpha_1 - \nu_1)}{2\beta_1\beta_p - \gamma^2} + \nu_p \ . \tag{8}$$

If α_p is less than or equal to $\underline{\alpha}_p$, private-label quality is so poor that demand for it will not exist. Note that $\underline{\alpha}_p$ is a strictly positive value. Further, the quality of the private label is bounded above by α_1, the quality of the national brand. Given these bounds, the retailer chooses the level of α_p so as to maximize her long-term profit:

$$\text{Maximize}_{\alpha_p} \quad \Pi_{\mathrm{R}} = m_p^*(\alpha_p)q_1^*(\alpha_p) + (p_p^*(\alpha_p) - v_p(\alpha_p))q_p^*(\alpha_p) \tag{9}$$

$$\text{Subject to} \qquad \underline{\alpha}_p \le \alpha_p \le \alpha_1 .$$

The exact expression of equation (9) is long and tedious, and is omitted here. Recall that we are assuming the private label's unit variable cost (v_p) to be an increasing function of α_p: a higher product quality can only be achieved at a higher average cost. The optimal solution depends on how the cost function is defined. If the cost increases fast and exponentially, the lowest value of α_p would be optimal. Otherwise, the behavior of the profit function depends on the assumed cost function. A number of studies use a simple quadratic function to obtain interior solutions for quality (i.e., Moorthy 1988; Tirole 1989; Motta 1993). We could also derive an "optimal" quality assuming a well-behaved cost function, but instead choose to examine more general implications in this paper.

Consider a limiting case in which increasing the private label's quality level is costless, so that v_p is a constant. Then the second order derivative of the retailer's profit with respect to α_p can be derived as

$$\frac{\partial^2 \Pi_R}{\partial \alpha_p{}^2} = \frac{1}{8}\left(\frac{3}{\beta_p} + \frac{\beta_1}{\beta_1\beta_p - \gamma^2}\right), \tag{10}$$

which is unambiguously positive and constant. Thus, with costless quality augmentation, the retailer's profit is a quadratic convex function of α_p. The minimum profit occurs at

$$\breve{\alpha}_p = \frac{\beta_p \gamma(\alpha_1 - v_1)}{4\beta_1\beta_p - 3\gamma^2} + v_p . \tag{11}$$

A brief comparison between equations (8) and (11) shows that $\underline{\alpha}_p > \breve{\alpha}_p$. This implies directly that all feasible values of α_p lie in the range where retail profit is increasing in α_p. Therefore, under the assumption that quality augmentation is costless, the optimal quality for the private label is $\alpha_p \to \alpha_1$: i.e., *as high as that of the national brand*. Figure 1 shows the shape of the retailer profit function. At first glance, this result seems trivial and uninteresting because quality improvement is assumed to be costless. In fact, a number of studies have used quadratic cost functions in order to obtain interior solutions for the lower-quality product (e.g., Motta 1993; Desai 2001). Our model would also result in a finite optimal value for private label quality when a quadratic cost function is used. However, we note that in either case, the implied solution is in direct contrast to the maximum differentiation result in the location theory literature (e.g., Shaked and Sutton 1982), in which the second brand is maximally differentiated in quality from the major brand (which here means it has the minimum quality level, $\underline{\alpha}_p$).[6] We formalize this result as follows:

Proposition 2: For a fixed value of the horizontal substitutability parameter γ, and costless private label (α_p) quality improvement, it is optimal for the retailer to seek minimum vertical differentiation from the national brand.

[Insert Figure 1 about here]

In addition, it can be shown that the national brand's equilibrium price is higher than that of the private label, due to double marginalization and the differences in variable costs:

Proposition 3: Even when the private label's quality is same as that of the national brand, the national brand's equilibrium price is higher than that of the private label.

Proof: See Technical Appendix.

Consider Propositions 2 and 3 in light of Table 1. For illustrative purposes, suppose the private label is horizontally undifferentiated from the national brand, as in the Ann Page – Campbell's case. Then Proposition 2 implies that the retailer's best strategy for the Ann Page brand is to increase its quality level to that of Campbell's; but Proposition 3 then implies that even so, we should still expect to see a retail price differential between the two brands, due to the degree of horizontal differentiation between them.

Effects of Horizontal Differentiation

An empirical study by Sayman, Hoch, and Raju (2002) found that a store brand can explicitly target a national brand by imitation (i.e., horizontal imitation via trade dress design), even if this has no effect on the perception of its quality (vertical differentiation). This implies that the two differentiation dimensions can be separately operated in some product classes. Assuming the quality level of the private label (α_p) is fixed, therefore, we now turn our attention to the role of the substitutability parameter γ. It is straightforward to show that the retailer's equilibrium profit from Table 2 decreases as γ goes up:

$$\frac{\partial \Pi_R^*}{\partial \gamma} = -\frac{(\beta_1(\alpha_p - \nu_p) - \gamma(\alpha_1 - \nu_1))(\beta_p(\alpha_1 - \nu_1) - \gamma(\alpha_p - \nu_p))}{8(\beta_1\beta_p - \gamma^2)^2} < 0. \qquad (12)$$

This means that the retailer's total profit is maximized at the lowest possible γ: hence the maximum degree of overall cross-product differentiation is optimal for the private label, in the absence of any quality changes. We formalize this as:

Proposition 4: When the substitutability parameter γ can be changed without changing α_p, the lowest possible γ is the optimal solution for the retailer. That is, maximum horizontal differentiation is optimal.

The intuition for this horizontal differentiation result is seen most clearly by considering the form of the utility function itself. The lower is γ, the higher is consumer utility, all other things constant, because consumers value variety; and higher utility levels lead to greater willingness to pay and greater consumption, all of which lead to higher retail profitability.[7] In short, if the retailer can reduce the extent to which the private label is viewed as interchangeable with the national brand, the marginal utility of consumption for *both* products is higher.

Comparing this result with those found in location models in the Economics literature, our finding of maximal horizontal differentiation is similar to that in Gabszewicz and Thisse's (1979) location modeling framework.[8] Our minimum vertical differentiation result is partially consistent with Vandenbosch and Weinberg (1995) (although the models are not directly comparable, as Vandenbosch and Weinberg look at a location model with two dimensions of vertical differentiation and no possibility of horizontal differentiation).

Consider this result in light of the example of Sears Kenmore appliances in Table 1. Many Kenmore appliances are in fact manufactured for Sears by General Electric (GE), and hence have virtually equal quality to GE appliances (hence, little vertical differentiation). Given the lack of vertical differentiation, Sear's best strategy for Kenmore is to differentiate it horizontally (e.g., in placement of control knobs and dials, or in colors and facades) from GE's refrigerators, while maintaining a quality level equal to GE and a positive price differential for GE over Kenmore.

Combined Effects

The above two analyses show that when the private label can be differentiated purely in the vertical dimension, minimum differentiation (i.e., the maximum possible level of α_p) is optimal from the retailer's point of view. When the private label can be differentiated purely in the horizontal dimension, maximum differentiation (i.e., the minimum possible level of γ) is optimal. Thus, we are tempted to conclude that in the combined situation, the private label should be vertically undifferentiated and horizontally differentiated, as in the Kenmore – GE example in Table 1. But does this mean, for instance, that President's Choice makes an error in minimally differentiating (both horizontally and vertically) its chocolate chip cookies from Chips Ahoy, the national brand? Not necessarily: our analysis shows that the result depends on how separable the two differentiation dimensions are in any particular product class. The tempting conclusion that low vertical differentiation and high horizontal differentiation are the optimal combination rests on the implicit assumption that these dimensions are in fact set independently of one another. We proceed to consider the impact of relaxing this implicit assumption here.

In the cookie example in Table 1, it is reasonable to suppose that a higher α_p can come at the expense of a higher γ as well. That is, a higher quality private label cookie can be also viewed as very similar horizontally to the national brand. This is bad news for the retailer, who would increase α_p but minimize the value of γ if she could. The strategic decision of the retailer therefore becomes how high to let α_p rise, which may depend on the extent to which this also raises the substitutability parameter γ. So logically, if a rise in α_p is accompanied by a rise in γ, the possibility exists that retailer profits could actually be harmed by an increase in α_p.

In general, the change in retail profit for a change in α_p is given by $\frac{d\Pi_R^*}{d\alpha_p} = \frac{\partial \Pi_R^*}{\partial \alpha_p} + \frac{\partial \Pi_R^*}{\partial \gamma}\frac{\partial \gamma}{\partial \alpha_p}$, where $\frac{\partial \Pi_R^*}{\partial \alpha_p} > 0$, $\frac{\partial \Pi_R^*}{\partial \gamma} < 0$, and $\frac{\partial \gamma}{\partial \alpha_p} > 0$. The derivative will be negative (implying it is *not* optimal to increase α_p to its maximally limiting value) if $\frac{\partial \gamma}{\partial \alpha_p} > -\frac{\partial \Pi_R^* / \partial \alpha_p}{\partial \Pi_R^* / \partial \gamma}$. The question then is whether this inequality is ever met, and if so, when. To examine this issue, we assume for the moment that differentiation is once again costless (since quality increases in the private label could always be costly enough to dissuade the retailer from undertaking them). Recall from equation (10) that the convexity of the retailer's profit function with respect to α_p is established by:

$$\frac{\partial^2 \Pi_R}{\partial {\alpha_p}^2} = \frac{1}{8}\left(\frac{3}{\beta_p} + \frac{\beta_1}{\beta_1\beta_p - \gamma^2}\right) > 0 .$$

Clearly, the convexity of the retail profit function with respect to α_p increases with increases in γ (that is, the derivative becomes greater in value when γ increases). This means equivalently that the more interchangeable are the private label and the national brand (the higher is γ), the greater is the marginal profit increase to be had by increasing product quality, α_p (assuming no incremental cost of doing so). From a utility perspective, this makes sense: a higher γ means a lower marginal utility of consuming either product, the more the other is consumed, and this decreases consumer willingness to pay and profitability. Marginal increases in product quality then have a greater impact on profitability.

Further, note that an increase in γ has three effects on the *position* of the equilibrium retail profit function as a function of α_p: (a) it shifts the *minimum* of the function *to the right* (i.e., to a higher value of α_p); (b) it causes the *minimum* of the retail profit function to occur at a *higher* level of retail profit; and (c) it causes maximum achievable retail profits (at $\alpha_p = \alpha_1$) to *drop* in value. We summarize these properties of the retail profit function in the following Lemma:

Lemma 1: Absent marginal cost impacts, the equilibrium retail profit function is not only convex in α_p, but exhibits the following properties: (i) its degree of convexity increases with increases in γ; (ii) increasing γ causes the *minimum* of the equilibrium retail profit function with respect to α_p to occur at a *higher* level of α_p; (iii) increasing γ causes the *minimum* of the equilibrium retail profit function with respect to α_p to occur at a *higher* level of equilibrium retail profit; and (iv) increasing γ *reduces* the maximum achievable level of equilibrium retail profit.

Proof: See Technical Appendix.

Figure 2 shows a set of representative equilibrium retail profit functions as functions of α_p, for different values of γ that illustrate these four properties (recall that the feasible values of α_p for each curve are ones in the upward-sloping portion of the curve; the entire curve is shown merely for illustrative purposes). As the set of curves shows, whether or not retail profit increases when the retailer invests in increasing the quality of its private label (α_p) depends on how much this increase affects the overall differentiation between the products, represented by increasing γ, and also depends on the starting value of γ. For example, consider the parameter values in Figure 2, with γ=.1 and α_p=0.8 as a starting point for the private label. In this situation, the private label's vertical quality is reasonably close to that of the national brand (which has α_1=1), and the products exhibit fairly low overall interchangeability, represented by the difference between γ and its maximum value of 0.3 (the value of β_1 in this example). Given these values, equilibrium retail profit equals 0.334953. Now, suppose that the retailer wishes to increase the quality of its private label. For these starting values, as long as the retailer can achieve a private label quality level of at least α_p=0.837543, it can increase its total profitability – even if this means that total interchangeability between the private label and the national brand rises to γ=.29, very close to its maximum permissible level given by the level of β_1 in this example. If it can increase quality to its maximum level of $\alpha_p = \alpha_1 = 1$ accompanied by a rise of γ to its maximum value of .29 as well, total retail profit rises to 0.484861, a net increase even given the loss due to a higher value of γ.

[Insert Figure 2 about here]

On the other hand, with the other parameter values as depicted in Figure 2, imagine a starting point of α_p=.94 and γ=0.05, generating a retail profit of 0.488714. The retailer cannot make herself better off by increasing α_p to its maximum value of 1 if this would cause γ to rise to 0.29, as this generates a profit of only 0.484861. A similar situation occurs if the starting point is α_p=.96 and γ=0.1, which generates a retail profit of 0.488701. These examples show that increasing the private-label quality level is unprofitable if (a) the retailer's starting point is a high-quality, highly horizontally differentiated product (i.e. a high value of α_p, but counterbalanced by horizontal differentiation sufficient to keep γ quite low), and (b) a further increase in the quality is accompanied by a large enough increase in overall interchangeability between the private label and the national brand. Under such circumstances, imitating the "trade dress" of the national brand would be a bad idea for the private-label retailer, for example. However, if the retailer's starting position for the private label is not so fortuitous, then increasing the quality of the private label is likely to be a profitable exercise, if it only induces an increase in γ without a corresponding increase in marginal production costs.

These examples show that the tradeoff between increasing private-label quality and associated increases in interchangeability have a strong impact on whether a retailer finds it profitable to improve private label quality. That is, when an increase in the private label's quality is accompanied by an increase in the overall interchangeability of the private label and the national brand (i.e., a higher γ), the optimal quality level is not always the maximum possible, even when this does not increase the retailer's marginal cost of production for the private label.

They further show that the lower is the starting value of γ and the higher is the starting value of α_p, the less likely this is to be a good investment. We formalize these results in the following Proposition:

Proposition 5: Suppose the vertical and horizontal dimensions are correlated. Increasing the quality of the private label to its maximum value is less likely to be profitable, the less is the initial vertical quality differentiation between the private label and the national brand (i.e., the higher is α_p), and the less is the initial interchangeability between the private label and the national brand (i.e., the lower is γ).

Note that the above calculations have treated the retailer kindly, in that they assume no increase in the marginal cost of production of the private label when α_p is increased. Proposition 5 points out that even with no increase in marginal cost, it may not be optimal to increase private label quality to its maximum possible level; but for this to be true, the initial positioning of the private label must be extraordinarily favorable in terms of α_p and γ. One might ask whether it is ever likely that we would observe such conditions in the real world; perhaps not, but it is also more realistic to assume that an increase in the quality of the private label, α_p, carries with it an increase in its marginal cost of production for the private label, ν_p. In the limit, it is reasonable to assume that when $\alpha_p=\alpha_1=1$, the level of ν_p should rise to equal ν_1 (the marginal cost of production of the national brand) as well as increasing the level of γ.

It is straightforward (and not surprising) to show that equilibrium retailer profits are unambiguously decreasing in ν_p. Thus, any increase in the marginal cost of production of the private label accompanying other parametric changes will worsen the profit picture for the retailer, relative to the cases discussed immediately above. An extension of the examples above suffices to show the pattern. Figure 3 shows two curves that are identical to those in Figure 2: one of retailer profit for γ=.1 and ν_p=.2, and one of retailer profit for γ=.29 and the same value of ν_p=.2. However, suppose that increasing private label quality to α_p=1 can only be achieved with an increase in γ to .29 *and* an increase in v_p to the level of v_1 (=.3). Then it is also relevant to plot the retail profit curve for γ=.29 and v_p=.3, which also appears in Figure 3.

If private label quality of α_p=1 means that both γ=.29 and v_p=.3, the resulting retail profit is 0.381178. If the private-label retailer starts out with a private label described by γ=.1 and v_p=.2, any starting value of α_p greater than or equal to 0.852687 makes it impossible for the retailer to improve its profit position by investing in increasing the vertical quality of its private label product. This contrasts with the situation we analyzed when discussing Figure 2 above, where we assumed that there was no marginal-cost penalty for increasing private label quality. In that case, where a private label quality of α_p=1 means that γ=.29 but that v_p remains at its starting value of 0.2, retailer profit is 0.484861; if the private-label retailer started out with a private label described by γ=.1 and v_p=.2, the starting value of α_p would have to be greater than or equal to 0.956453 for it to be impossible for the retailer to improve its profit position by investing in increasing the vertical quality of its private label product. This illustrates the impact of the marginal-cost penalty on the retailer's decision process in fixing the quality of its private label. While the qualitative nature of our conclusions does not change, the likelihood that the retailer will choose *not* to invest in a quality level close to that of the national brand is much greater when we take account of the marginal-cost effect of producing a higher-quality private label product.

[Insert Figure 3 about here]

We summarize this insight as:

Proposition 6: When an increase in the private label's quality is accompanied by both an increase in its overall interchangeability with the national brand and an increase in the marginal cost of production of the private label, the optimal quality level is not always the maximum possible α_p. Increasing the quality of the private label to its maximum value is less likely to be profitable, the less is the initial vertical quality differentiation between the private label and the national brand (i.e., the higher is α_p), and the less is the initial interchangeability between the private label and the national brand (i.e., the lower is γ).

This result of course does not imply that the private-label retailer rarely or never finds it profitable to invest in increasing private-label quality, but does provide predictions about underlying conditions on consumer utility for the products that make it more or less likely to occur. Clearly, cost is a critical factor that keeps retailers from investing more in their private labels; one business press article states, "Many retailers balk at marketing their store brands, envisioning expensive ad campaigns aimed at building brand equity…. No matter how much [retailers] talk about reinvesting three percent back into the [private label] brand, [retailers] don't do it" (Wellman 1997). This quote suggests that even beyond the marginal-cost increases we examine here, there may be significant fixed costs of investing in private-label branding which further deter the retailer from seeking to imitate the national brand.

TWO NATIONAL BRANDS CASE

In this section, we extend the simple demand model to the case where there are two national brands. These national brands can have various degrees of horizontal differentiation. In some product categories (such as canned soups), the two leading national brands (Campbell's and Progresso) are barely differentiated horizontally. On the other hand, in the analgesics category, products are widely differentiated even with the same ingredients. For example, both Tylenol and Midol have acetaminophen as their primary ingredient. However, Tylenol is positioned as the general pain reliever, while Midol is strongly entrenched for menstrual cramps. In the latter category, the retailer's decision is to position its private label between the two claims, and Sayman et al. (2002) find that the private label should imitate the stronger brand (i.e., Tylenol). However, in the former case, the private label soup may have another option than imitating the two national brands—it may differentiate "outside the space" between the two national brands. These two scenarios are examined separately in the following two subsections.

Case 1: National Brands are Sufficiently Differentiated

We first generalize our base demand model (2) and (3) assuming the two national brands are differentiated. Later we examine the case where these national brands are horizontally undifferentiated. The consumer utility function (1) can be extended as follows by adding the second national brand (denoted by subscript 2):

$$\begin{aligned} U(q_1,q_2,q_p) &= (\alpha_1 - p_1)q_1 + (\alpha_2 - p_2)q_2 + (\alpha_p - p_p)q_p \\ &\quad - (1/2)(\beta_1 q_1^2 + \beta_2 q_2^2 + \beta_p q_p^2 + 2\gamma_{12}q_1q_2 + 2\gamma_{1p}q_1q_p + 2\gamma_{2p}q_2q_p) \end{aligned}, \tag{13}$$

where β s and γ s have the same interpretation as in the base case. Solving the three first-order conditions of utility maximization for q_1, q_2, and q_3, we obtain the following demand system:

$$q_1 = \frac{1}{T}[B_1 - (\beta_2\beta_p - \gamma_{2p}^2)p_1 + (\beta_p\gamma_{12} - \gamma_{1p}\gamma_{2p})p_2 + (\beta_2\gamma_{1p} - \gamma_{12}\gamma_{2p})p_p], \tag{14}$$

$$q_2 = \frac{1}{T}[B_2 + (\beta_p\gamma_{12} - \gamma_{1p}\gamma_{2p})p_1 - (\beta_1\beta_p - \gamma_{1p}^2)p_2 + (\beta_1\gamma_{2p} - \gamma_{12}\gamma_{1p})p_p], \text{ and} \tag{15}$$

$$q_3 = \frac{1}{T}[B_3 + (\beta_2\gamma_{1p} - \gamma_{12}\gamma_{2p})p_1 + (\beta_1\gamma_{2p} - \gamma_{12}\gamma_{1p})p_2 - (\beta_1\beta_2 - \gamma_{12}^2)p_p], \tag{16}$$

where

$$T = \beta_1\beta_2\beta_p + 2\gamma_{12}\gamma_{1p}\gamma_{2p} - \beta_1\gamma_{2p}^2 - \beta_1\gamma_{1p}^2 - \beta_1\gamma_{12}^2,$$

$$B_1 = \alpha_1(\beta_2\beta_p - \gamma_{1p}^2) - \alpha_2(\beta_p\gamma_{12} - \gamma_{1p}\gamma_{2p}) - \alpha_p(\beta_2\gamma_{1p} - \gamma_{12}\gamma_{2p}),$$

$$B_2 = -\alpha_1(\beta_p\gamma_{12} - \gamma_{1p}\gamma_{2p}) + \alpha_2(\beta_1\beta_p - \gamma_{1p}^2) - \alpha_p(\beta_1\gamma_{2p} - \gamma_{12}\gamma_{1p}),$$

$$B_3 = -\alpha_1(\beta_2\gamma_{1p} - \gamma_{12}\gamma_{2p}) - \alpha_2(\beta_1\gamma_{2p} - \gamma_{12}\gamma_{1p}) + \alpha_p(\beta_1\beta_2 - \gamma_{12}^2).$$

For the demand coefficients to have "correct" signs and for the second order conditions to be satisfied, we require $\beta_i \geq \gamma_{jk}$ for all $i, j, k = 1,2, p$, and $\beta_p \geq \beta_i$ for $i = 1,2$. That is, the rates of decrease in own-marginal utilities are greater than cross-marginal utilities, and satiation occurs faster when consuming the private label than when consuming a national brand.

The retailer's profit maximization problem with respect to the retail prices is equivalent to equation (4) but with three brands. The retailer's reaction functions are found as:

$$p_1 = (w_1 + \alpha_1)/2, \quad p_2 = (w_2 + \alpha_2)/2, \quad p_p = (\alpha_p + \nu_p)/2. \tag{17}$$

As in the base case (equation 4), the private label's price is independent of the quality levels of the national brands.

We obtained the equilibrium solution for the Nash game between the two national brands that act as Stackelberg leaders against the private label results, but they are not presented here because the equations are long and not immediately meaningful. (Interested readers can find the solution in the Technical Appendix.) Instead, we simplify the solution by normalizing $\beta_1 = \beta_2 = 1$. Note that $\beta_p \geq \beta_i = 1$. Without loss of generality, we assume national brand 1 is the higher quality brand: $\alpha_1 \geq \alpha_2 \geq \alpha_p$. Let $x \in [0,1]$ denote the level of vertical differentiation such that $\alpha_p = x\alpha_2$. A larger x represents *less* vertical differentiation. We also assume the variable cost of private label is normalized to zero ($v_p = 0$). In addition, since we are focusing on the case of intermediate differentiation for now, we restrict the horizontal differentiation parameter to be such that $\gamma_{12} = \gamma$, and γ_{1p} and γ_{2p} having the following relationship: When the private label imitates national brand 1, $\gamma_{1p} = \beta = 1$ and $\gamma_{2p} = \gamma$, and vice versa. The linear rule implied is: $\gamma_{1p} = \beta - k$ and $\gamma_{2p} = \gamma + k$, where $k \in [0,(\beta - \gamma)]$. When $k = 0$, the private label is horizontally imitating national brand 1; and when $k = 1 - \gamma$, it imitates national brand 2.

As in the base case above, we are interested in finding the behavior of retailer profit as a function of the two types of differentiation. However, the function is polynomial and very long even with the simplification. Therefore, we resort to a numerical approach to evaluate its properties. An extensive computation reveals that the general shape of the profit function is consistent, and it is convex in both dimensions within the parameter domain. Figure 4 presents a typical profit surface. Note that x varies between 0 and 1, and indicates the level of vertical differentiation ($x = 1$ means the private label has the same quality level as the lower-quality national brand). k represents the horizontal location of the private label between the two national brands ($k = 0$ means the private label horizontally imitates national brand 1).

[Insert Figure 4 and 5 about here]

In interpreting Figure 4, we need to focus on the parameter region in which the private label's demand is positive (Figure 5). When its quality is low, the private label's demand is negative (with negative price), leading to a false positive profit level. Combining insights from the two figures, we find that it is optimal for the retailer to position the private label (a) with minimum vertical differentiation from national brand 2 (x=1), and (b) with minimum horizontal differentiation from the higher quality national brand's trade dress (k=0). The minimum vertical differentiation result contradicts those from the location models in the economics literature (e.g., Shaked and Sutton 1982; Tirole 1989). Note, however, that this latter result is obtained without considering the cost of increasing the quality of the private label. When it is costly to increase private label quality, minimum differentiation will not always be the best decision.

If the quality levels of the two national brands are equal ($\alpha_1 = \alpha_2$) and they are substantially horizontally differentiated, imitating either brand is optimal for the private label. However, when the national brands are allowed to be asymmetric (different α levels), the private label is better off imitating the higher quality brand. This is consistent with Sayman et al.'s (2002) result. Positioning in between is never an optimal solution.

Case 2: The Two National Brands are Horizontally Undifferentiated

When the two national brands lack horizontal differentiation, the private label might have another option for its horizontal positioning – horizontally differentiated from both national brands.[9] The car battery market provides an example of this phenomenon. Two major national brands, ACDelco and Exide, are virtually undifferentiated; would the Sears private label battery, DieHard, be better off imitating them? We in fact observe that the private label differentiates itself as the "tough" battery. In this section, we modify our model to the special case of two undifferentiated national brands.

In our demand functions (14-16), having undifferentiated national brands would mean that $\gamma_{1P} = \gamma_{2P} = \gamma_P$ from our private label's point of view: that is, the private label is equally horizontally differentiated from both national brands. Further, a lack of horizontal differentiation between the two national brands means that consuming a unit of either one has an equal effect on the marginal utility of consuming an incremental unit of one of the brands. Mathematically, this implies that the cross-price demand parameter is essentially the same as the own-price demand parameter; but to insure that the problem has an interior solution, we need to introduce a small number ε such that $\gamma_{12} = \beta - \varepsilon$, where we recollect that γ_{12} is the cross-product substitutability parameter between the two national brands.

We derive the equilibrium profits of the retailer and manufacturer, as well as equilibrium prices and quantities, with these assumptions. While they can be straightforwardly derived, the expressions are too unwieldy to present here. As in the discussion of differentiated national brands above, we therefore assume $\beta_1 = \beta_2 = \beta = 1$. As above, $\beta_P \geq \beta = 1$. Hence, $\gamma_{12} = 1 - \varepsilon$. For simplicity, we first examine the case where the two national brands are characterized by the same vertical quality: $\alpha_1 = \alpha_2 = \alpha \geq \alpha_P$. Let $x \in [0,1]$ denote the level of vertical differentiation such that $\alpha_P = x\alpha$. An analysis of the equilibrium demand for the store brand shows that store-brand sales are positive only when $\gamma_P < x$, a condition we need to impose to evaluate realistic situations where the store brand in fact competes with the two similarly-positioned national brands. Intuitively, this condition implies that if the store brand is "too inferior" in quality to the national brands (i.e., if $x < \gamma_P$), store-brand demand will be negative. Alternatively, the condition implies that the relative quality of the store brand as compared to national brands places a limit on how horizontally differentiated the store brand can be from the national brands, and still garner positive sales; the higher is the store brand's quality, the more horizontally differentiated it can be and still survive in the market.

Without loss of generality, we therefore restate γ_p as $\gamma_P = y \cdot x$, with $y \in [0,1]$. Equilibrium retailer profits in this situation, as ε approaches zero, are given by $\dfrac{\alpha^2\left[\beta_P + x^2(1-2y)\right]}{4\left(\beta_P - x^2 y^2\right)}$. The question of what is the optimal degree of horizontal differentiation of the store brand from the national brands translates in this context to a choice of the value of y (in the permitted range of $0 \leq y \leq 1$) that maximizes these equilibrium retailer profits. It is straightforward to show that this profit expression is decreasing in y: the optimal value of y is thus zero, implying maximal horizontal differentiation of the store brand from the national brands. We can also easily show that (if it were costless) the retailer would choose *minimal* vertical differentiation of the store brand from the national brands, that is, $x = 1$. Finally, it is also easy to show that in this situation, maximal horizontal store brand differentiation also maximizes unit sales of the store brand.

Interestingly, therefore, the nature of competitive positioning between the national brands affects the relative position the retailer wishes to establish for its store brand. In the case of horizontal differentiation between the national brands, the retailer optimally positions the store brand at the same horizontal position as one or the other of the national brands – minimizing horizontal differentiation with one of the national brands (similar to the result in Sayman, Hoch and Raju 2002). But here, when the national brands themselves are undifferentiated, it is no longer in the retailer's best interest to choose a horizontal position equal to those of the national brands; instead, retail profit is maximized by maximally horizontally differentiating the store brand from the national brands. Neither Sayman et al. (2002) nor Du et al. (2004) derive this result, because they do not consider the possibility that the private label horizontally differentiates from two horizontally *un*differentiated national brands. Thus, in a multi-national-brand world, private-label positioning decisions optimally take account of the existing national-brand competition.

We can also extend our results to a case where the two national brands, while *horizontally* undifferentiated, are *vertically* differentiated, with $\alpha_1 > \alpha_2 \geq \alpha_P$. We show a numerical example of this more complicated case in the Technical Appendix to illustrate that (a) the more quality differences there are between the two national brands, the more stringent is the condition on minimum quality of the store brand to guarantee positive store-brand sales; and (b) it is still optimal for the retailer to maximize the horizontal differentiation of the store brand from the national brands (subject to positive sales of the store brand), as above. Thus, our results on the optimality of maximum horizontal differentiation of the store brand, given national brands that lack horizontal differentiation, holds regardless of the quality differences between the national brands.

In sum, our analysis of the case of two national brands competing with the private label shows that optimal positioning of the store brand should take into account the relative horizontal positions of the national brands. When national brands are significantly differentiated only in the horizontal dimension, the private label can horizontally position equivalently to one of the national brands. If the national brands are of different quality, the retailer horizontally positions the private label equivalently to the stronger national brand. But when national brands are not differentiated horizontally, the results are quite different: the retailer optimally *maximizes* the horizontal differentiation between the private label and the national brands.

IMPACT OF PRIVATE-LABEL POSITIONING DECISION ON NATIONAL BRAND PROFITS

Our main focus in this paper is on the retailer's decision of how to position the private label versus the national brand. However, it is interesting to examine the impact of these decisions on the national-brand manufacturer as well. One can imagine a longer time horizon than we model here, where a national-brand manufacturer, knowing the retailer's strategic and tactical incentives to position the private label and to price both products, would optimally invest in its own horizontal and/or vertical differentiation efforts through time. While modeling this more extensive game is beyond the scope of this paper, it is instructive to think about the impact on the manufacturer of these retailer actions.

Examining the manufacturer's equilibrium profit expression in the single national brand case (Table 2), it can be shown that the manufacturer's profit is convex in the value of γ: decreasing in γ at its low initial values, then eventually increasing with further increases in γ. The same convexity can be observed in the two undifferentiated national brands case. Further analysis reveals that the national brand manufacturer, like the retailer, benefits from facing a private label that is maximally horizontally differentiated from the national brand. Further, it is clear that the manufacturer's equilibrium profit is *decreasing* in α_p, the quality of the private label, holding the manufacturer's national brand quality (α_1) constant (whether there are two or just one national brands in the marketplace).[10] Thus, like the retailer, the national brand manufacturer benefits from products that are horizontally differentiated; but unlike the retailer, the national brand manufacturer also benefits from competing with a lower-quality private label product, i.e., facing maximal vertical differentiation as well.

The implications are somewhat daunting for the national brand manufacturer, particularly when we take into account the likely positive correlation (rather than independence) between increases in private label *quality* and increases in the γ parameter (that is, decreases in horizontal differentiation between the private label and national brand). The national brand manufacturer is harmed by a strategic retailer that seeks to increase the quality of the private label – *particularly* if this quality increase also comes at the expense of a higher level of γ! Such a situation implies a double penalty to manufacturer-level profits.

Recognizing these implications, a strategically oriented manufacturer is likely to persist in efforts to both increase its quality and to increase its horizontal differentiation from the private label through time. In the differentiated national brands case, on the other hand, the private label is more likely to imitate the stronger brand in both horizontal and vertical dimensions and share the segment revenue. Depending on the comparative strength and horizontal differentiation possibilities, therefore, it is conceivable that a national brand might actively seek the position of the second quality, rather than the top quality.

DISCUSSION OF RESULTS AND FUTURE RESEARCH DIRECTIONS

The majority of the location theory literature suggests that maximum differentiation in both vertical and horizontal dimension is the equilibrium result. However, the push to make private label products "better," which decreases differentiation, is not surprising, given our analysis. Even when a successful private label attacks the national brand by creating an offering of virtually equal quality, a retailer's profitability can increase if this product improvement is not too costly and/or is accompanied by persistent horizontal differentiation. Nevertheless, there are hints that retailers perceive extensive private-label marketing campaigns – that could increase the quality perception of their store brands – to be very expensive. Our analysis suggests that a costly campaign, combined with an initially reasonably favorable private label product position, may not in fact be a profitable move by the retailer.

Our analysis shows that optimal differentiation strategies by the private label depend on several underlying factors. When consumer tastes are diverse (the location model), maximum vertical differentiation would be optimal (breakfast cereal). When consumers are relatively homogeneous (representative consumer utility model), minimum vertical differentiation is indicated (canned soup). When the national brands are substantially differentiated horizontally, the private label is generally better off imitating the stronger national brand (analgesics). When the national brands lack clear horizontal differentiation, the private label is better off taking a distinct position (car batteries).

The national-brand manufacturer, meanwhile, does best in a retail environment where his brand is "king" in quality, and is highly horizontally differentiated from the private label. National brands thus stand to be hurt by strategic moves on the part of retailers to improve the quality of their private labels. On a larger level, the rational response of national brand manufacturers is to continually increase the quality of their national brands, find new methods of horizontal differentiation (such as novel packaging, sizes, and product forms), or both.

Finally, our research shows that the *nature* of differentiation matters in private label – national brand competition. Product substitutability is not a unidimensional concept; a focus on vertical or quality investments alone obscures the importance of horizontal differentiation in maintaining a vibrant product category. Our utility-based demand model, allowing for the possibility of natural market expansion with greater product variety as well as greater product quality, highlights the multiple positioning opportunities open to retailers as they seek to optimize their investments in private labels, as well as to manufacturers seeking to preserve their national-brand dominance in the market.

The focus of this research is on the positioning decision of the retailer that manages both a national brand and a private label from an analytic and predictive point of view. Our research results suggest testable implications for future research, regarding the most likely combinations of positioning one might observe in national brand-private label competition, and the most likely conditions under which one would observe investments in improving the private label's quality. We also focus on a manufacturer-retailer interaction where the retailer is assumed to be a category manager. A different, but interesting, research question is what incentives such a retailer has to use category management rather than individual product management within the category to position and price its products. This research question could provide some interesting insights not only into the retailer's decisions, but also into the manufacturer's optimal defensive national brand positioning strategies, given that it sells through a retailer that either does or does not practice category management.

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Table 1. Two Dimensions of NB/PL Differentiation

<table>
<tr><td></td><td></td><td colspan="2">Horizontal</td></tr>
<tr><td></td><td></td><td>Undifferentiated</td><td>Differentiated</td></tr>
<tr><td rowspan="2">Vertical</td><td>Undifferentiated</td><td><u>Cookies</u>
Chip's Ahoy
vs.
President's Choice</td><td><u>Refrigerators</u>
GE
vs.
Kenmore</td></tr>
<tr><td>Differentiated</td><td><u>Canned soups</u>
Campbell's
vs.
Ann Page</td><td><u>Analgesics</u>
Tylenol
vs.
Acetaminophen</td></tr>
</table>

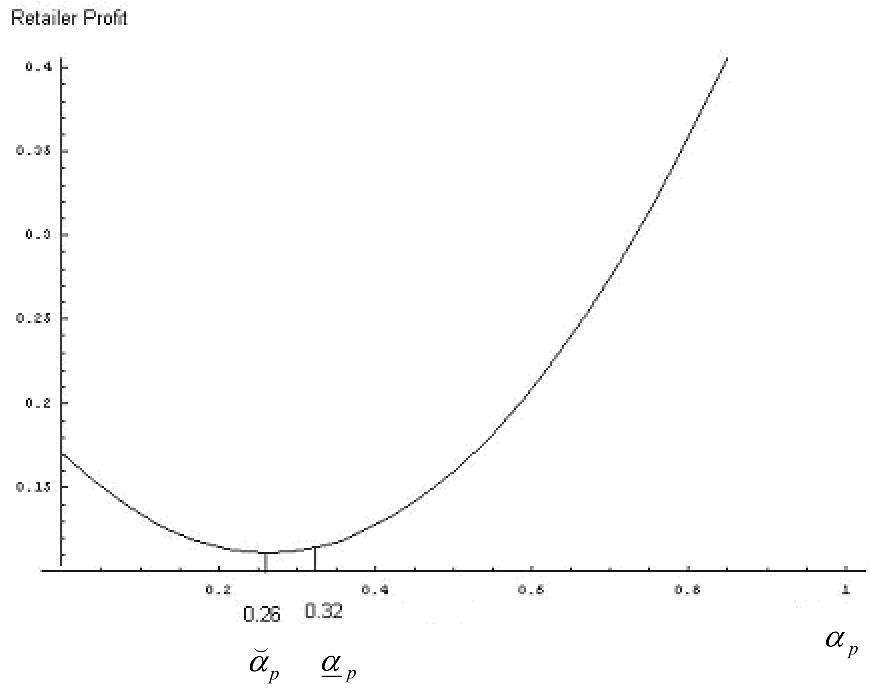
Table 2. Equilibrium Values (Product-Line Pricing at Retail)

	Results for Manufacturer and/or National Brand	Results for Retailer and/or Private Label
Wholesale price, w_1^* and retail margin, m_1^* for the NB	$\frac{\beta_p(\alpha_1+\nu_1)-\gamma(\alpha_p-\nu_p)}{2\beta_p}$	$\frac{\beta_p(\alpha_1-\nu_1)+\gamma(\alpha_p-\nu_p)}{4\beta_p}$
Retail prices, p_1^* and p_p^*	$\frac{\beta_p(3\alpha_1+\nu_1)-\gamma(\alpha_p-\nu_p)}{4\beta_p}$	$\frac{\alpha_p+\nu_p}{2}$
Quantities, q_1^* and q_p^*	$\frac{\beta_p(\alpha_1-\nu_1)-\gamma(\alpha_p-\nu_p)}{4(\beta_1\beta_p-\gamma^2)}$	$\frac{2\beta_1\beta_p(\alpha_p-\nu_p)-\gamma^2(\alpha_p-\nu_p)-\beta_p\gamma(\alpha_1-\nu_1)}{4\beta_p(\beta_1\beta_p-\gamma^2)}$
Profits of NB mfgr., Π_M^*, and retailer, Π_R^*	$\frac{(\beta_p(\alpha_1-\nu_1)-\gamma(\alpha_p-\nu_p))^2}{8\beta_p(\beta_1\beta_p-\gamma^2)}$	$\frac{4\beta_1\beta_p(\alpha_p-\nu_p)^2+Y}{16\beta_p(\beta_1\beta_p-\gamma^2)}$ (*)

(*) $Y=[\beta_p(\alpha_1-\nu_1)+\gamma(\alpha_p-\nu_p)][\beta_p(\alpha_1-\nu_1)-3\gamma(\alpha_p-\nu_p)]$; retailer profit function is the sum of retail profits across the private label and the national brand.

Figure 1: Retailer Profit as a Function of α_p

(Parameter values used are: $\alpha_1 = 1, \beta_1 = 0.3, \beta_p = 0.33, \gamma = 0.1, \nu_1 = 0.3, \nu_p = 0.2$)



Note: the minimum of the retail profit function for the parameters assumed above occurs at $\alpha_p = 0.26$, while the feasible range of α_p is $\alpha_p \in [0.32, 1]$. The profit-maximizing value of α_p is therefore α_p=1.

Figure 2: Retailer Profit as a Function of α_p for Various Values of γ

(Parameter values used are: $\alpha_1 = 1, \beta_1 = 0.3,\ \beta_p = 0.33,\ \nu_1 = 0.3,\ \nu_p = 0.2$)

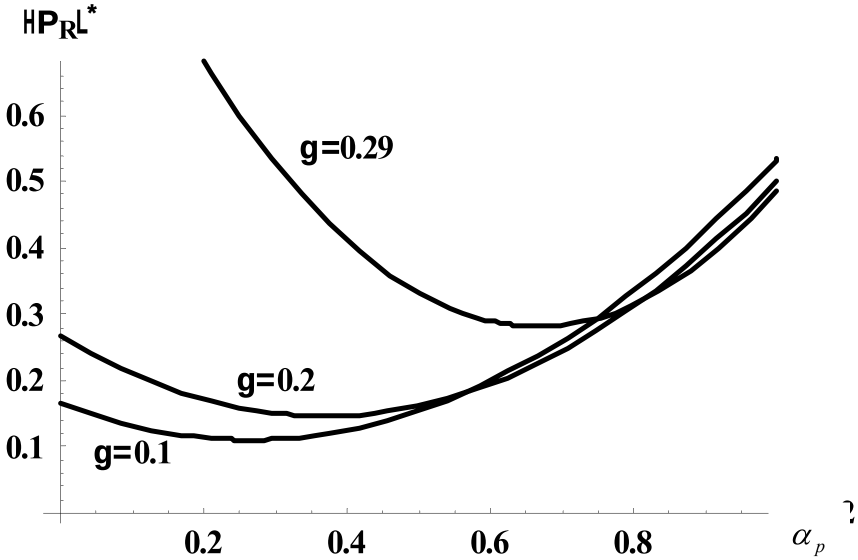


Figure 3: Retailer Profit as a Function of α_p for Various Values of γ and ν_p

(Parameter values used are: $\alpha_p = 1, \beta_1 = 0.3,\ \beta_p = 0.33,\ \nu_1 = 0.3$)

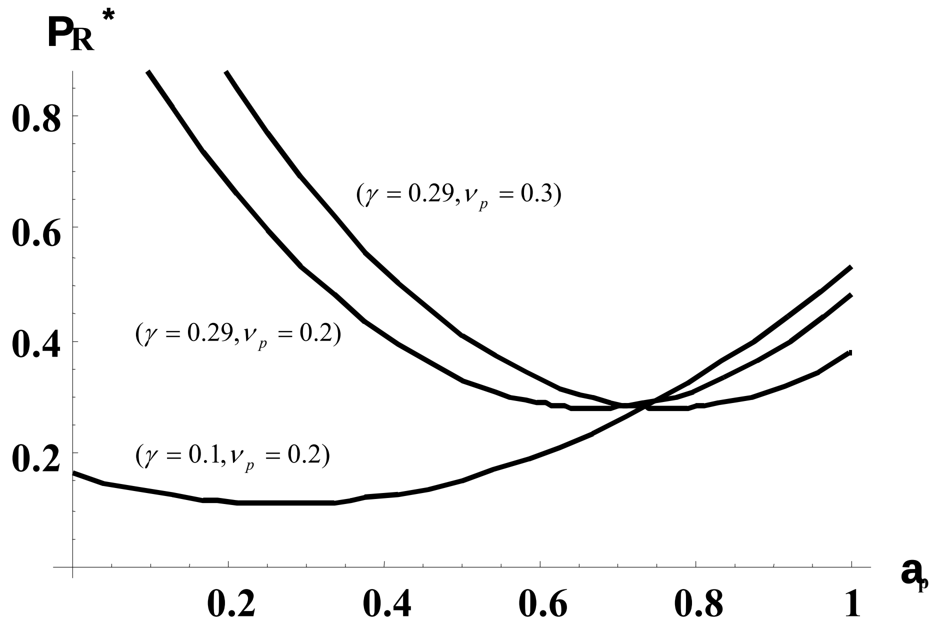
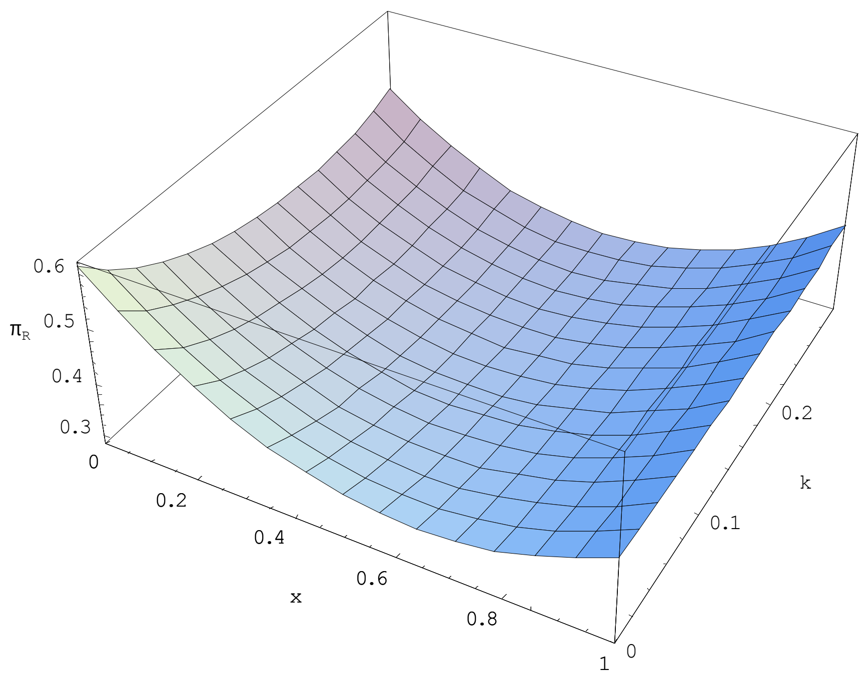


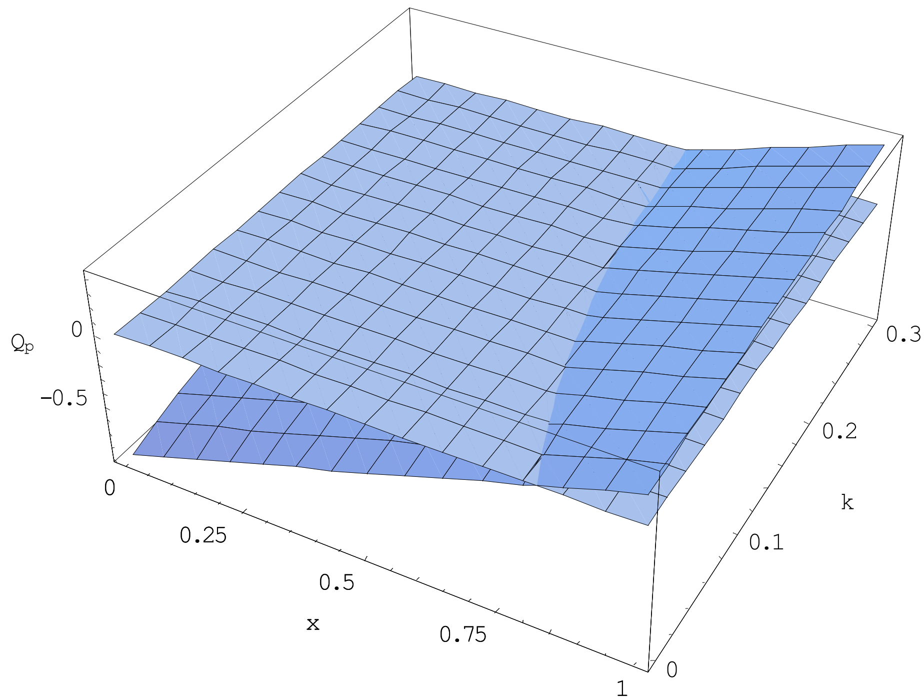
Figure 4: Retailer Profit in Vertical and Horizontal Differentiations

(Parameter values used are: $\alpha_1 = 1.5, \alpha_2 = 1, \beta = 1, \beta_p = 1.5, \gamma = 0.7$)



Note: see Figure 5 and its Note for the portion of this retailer profit curve associated with strictly positive sales of the private label. We show the entire function here for completeness, but only $\{x,k\}$ pairs leading to non-negative sales of the private label are of interest.

Figure 5: Parameter Region for Positive Private Label Demand



Note: Two surfaces are represented in this space. The lighter surface is the Q_P=0 plane. The darker surface shows actual values of Q_P, for any {*x,k*} pair. Only the part of that surface above the Q_P=0 plane is relevant for our investigations: *x* must be high enough, given *k*, to insure positive sales of the private label. Thus, only these positive-Q_P {*x,k*} pairs are relevant in Figure 4 above.

Footnotes

[1] One of the authors met and interviewed the son of the flavor chemist for A&P's private label products. This unfortunate man was forced to eat Ann Page private label products, including canned soups, throughout his childhood. He readily admitted that the Ann Page brand was significantly worse than the national brand competitors. This depiction of the private label as lower quality than the national brand is consistent with standard representations in the literature as well; see, for example, Soberman and Parker (2004).

[2] Several papers including Blattberg and Wisniewski (1989) and Sethuraman, Srinivasan, and Kim (1999) document the existence of this asymmetric price effects between the two quality tier products. Actual modeling of this asymmetry requires a nondifferentiable demand function that is kinked at every price points, and developing such a model is out of scope of the current study.

[3] We assume that $\beta_P > \beta_1 > \gamma$ in the analysis that follows.

[4] The Technical Appendix shows that the retailer profit function is concave.

[5] This general characteristic remains true for all other linear demand functions under product-line pricing. In contrast, when the retailer employs brand-centered pricing rather than product-line pricing, the optimal prices require knowledge of all parameters: $m_1 = \dfrac{(2\beta_1\beta_p - \gamma^2)(\alpha_1 - w_1) - \beta_1\gamma(\alpha_p - \nu_p)}{4\beta_1\beta_p - \gamma^2}$ and

$$p_p = \frac{2\beta_1\beta_p(\alpha_p + \nu_p) - \beta_p\gamma(\alpha_1 - w_1) - \alpha_1\gamma^2}{4\beta_1\beta_p - \gamma^2}.$$

[6] However, as will be discussed later (Proposition 5), when vertical and horizontal dimensions are correlated, the minimum quality differentiation result is not always optimal even when quality augmentation is costless.

[7] The change in total category demand (the sum of national brand and private label unit sales) with respect to a change in γ is: $\dfrac{-(\alpha_p - \nu_p)(\beta_1\beta_p - 2\beta_1\gamma + \gamma^2) - (\alpha_1 - \nu_1)(\beta_1\beta_p - 2\beta_p\gamma + \gamma^2)}{4\beta_p(\beta_1\beta_p - \gamma^2)}$, which is negative in all but the most unusual circumstances (for this derivative to be positive would require that the private label's quality is extremely low, so that the first term in the numerator is virtually zero, while also requiring that γ approach its maximum value of β_1, meaning that vertical differentiation is maximized, and yet total interchangeability is extremely high – not a reasonable concatenation of circumstances). Thus, the derivative is negative, meaning that the category does indeed expand in size when products become more differentiated. This is sensible given our utility framework, which posits that consumers value variety and will expand consumption when variety in the category increases.

[8] See also Economides (1986) for a summary of differentiation results that come out of location models, depending on their demand assumptions.

[9] This option is not considered by Sayman et. al (2002) or by Du et. al. (2004), but is easily handled in our utility framework.

[10] Please see the Technical Appendix for details of these calculations.