Channel Coordination with Price-Quality Sensitive Demand and Concave Transportation Cost
(Working paper of Choi, Lei, Wang, and CX Fan, 2004)

We analyze the impact of channel coordination on supply chain profitability for a single-product supply process involving a supplier, a buyer and a transporter with concave transportation cost functions. The buyer purchases the product from the supplier and then sells in the market. The market demand is assumed to be sensitive to the buyer’s selling price and the supplier’s (or manufacturer’s) product quality. We establish the dominance relationship between the profitability achieved by a totally centralized coordination and the sum of individual partner’s maximum profitability in a decentralized business environment. The effect of transporter’s coordination is analyzed. Policies to jointly optimize the buyer’s market selling price, the supplier’s quality level, and the shipping quantities handled by the transporter are proposed. Empirical observations that show the improvement on supply chain profitability by using the optimal policies assuming a market demand function that is decreasingly convex to the buyer’s (retailer’s) selling price and increasingly linear to the supplier’s (manufacturer’s) product quality are reported.

1. Introduction

As supply chain management captures the attention of more and more top level executives, companies are changing their ways of interacting, collaborating, sharing the information, and making decisions (O’Reilly, 2002). One important category of business decisions influenced by such changes is channel coordination policies that guide collaborative operations of partners involved in a supply chain process. Effective supply chain management necessitates a strong collaboration of participating partners and a solid implementation of strategies to optimize the total profitability of the chain as a whole entity. Among numerous examples of practical needs for supply chain coordination is the operational problems handled by British Airway Catering (BAC). BAC is responsible for delivering about 44 millions of meals per year prepared by hundreds of third party

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catering partners. Just in terms of London Kitchen alone, about 250 tons of chicken and 73 tons of eggs are prepared for BAC on annual basis. Together with finished meals, BAC also manages to purchase many other non-food items such as crockery, glassware, plastics, blankets, non-perishable dry foods, and others. Every time a jumbo jet takes off, about 40,000 items are pulled through the BAC supply chain. Any lack of coordination and integration can result in a serious problem in British Airway operations, either a delay in meeting passenger needs, overstock in expensive airport inventories, huge cost due to perishable foods, and loss in profitability (Christopher, 1998).

Buyer-supplier coordination has received a significant attention of researchers during the past two decades. Primary results in this regard can be found in the work by Goyal (1976, 1988), Monahan (1984), Lee and Rosenblatt (1986), Banerjee (1986), Goyal and Gupta (1989), Benjamin(1990), Anupindi and Akella (1993), Kohli and Park (1994), Lau and Lau(1994), Weng (1995,1997), and several of others, etc. These initial work have produced many valuable insights into the development of mechanisms to ensure the implementation of coordination policies in practice, and have built a foundation for later studies in the area. There have also been several important extensions recently. One is by Corbett and Groote (2000). They studied the optimal quantity discount policies under asymmetric information about the buyer’s cost structure, and compared the policy to the situation where the supplier has full information. Cheung and Lee (2002) analyzed the joint impact of shipment coordination strategy (between buyer and supplier) and stock rebalancing strategy to maximize the joint profit of supplier and buyer. The work is a very first one to study the joint effect of the two major supply chain strategies. Chen, Federgruen and Zheng (2001) generalized existing channel coordination models with identical buyers and developed effective mechanisms for managing a distribution process involving one supplier and multiple non-identical buyers. There have also been a number of papers published for the channel coordination with stochastic demand. A comprehensive review for this area can be found in the work of Cachon (2001).
In this study, we aim to add another extension to the area of buyer-supplier coordination by explicitly considering in our model 1) a market demand that is sensitive to both retailer’s (buyer’s) selling price and supplier’s product quality; and 2) a third party logistics partner with a non-negligible concave operation cost. In particular, we extend Kevin Weng’s two-partner (i.e., buyer-supplier) coordination model (Weng, 1995) into a more general case where the involvement of a logistics partner deprives the optimality of the known two-partner (buyer and supplier) economic ordering quantity. Our study tries to find answers to the following questions: what are the optimal coordination policies that jointly maximize the total supply chain profitability when the roles of supplier, buyer and transporter are equally important in the business ? what is the optimal trade off between the product quality and the operational spending on quality given that the profitability increases as product quality increases and decreases as the spending on quality goes beyond certain level ? and what are the impact of a centralized coordination on supply chain profitability as compared to that may be possibly achieved by an optimized but decentralized coordination ?

It is important to point out that there are many practical situations where the strategic partnership is established between a single supplier and a single buyer with the transportation outsourced to a third party logistics partner. For example, Logan Aluminum, a leading supplier of aluminum to the beverage can industry, has a large range of customers of various aluminum flatbed products, including the contracts with single customer for highly customized product. The third party logistics partner of Logan, who is responsible for shipping 1,000,000 pounds of aluminum coils every week is CRST International (John Smith, 2002). Such relationship has been very common between small manufacturers and their contracted sales and logistics partners. There are hundreds of third party logistics partners (transporters) on the market for providing the transportation services for supplier-buyer contracts. Examples of such transporters include Elite International for chemical, petrochemical and food shippers, Pilot Air Freight for safety window film manufacturers, Menlo Logistics for Sears and its contracted manufacturers, and Penske Trucking for Whirlpool and home appliances retailers.
In section 2 of the paper, we develop the profitability functions for individual partners involved in a supply chain process. We prove that the yearly profit of the transporter can be improved, even in a decentralized/independent business environment, if his/her fixed and variable operation cost can be incorporated into the buyer’s consideration on ordering quantities. We also discuss the conditions required to achieve this improvement. In section 3, we analyze joint optimal policies for all the three partners that maximize the overall (total) profitability of a supply chain. We show that the willingness of the transporter to coordinate and to share the profitability with other partners have a significant impact on total supply chain profitability. In section 4, we derive optimal selling prices, quality level, and shipping quantity under a decentralized, and a totally centralized, business environment, respectively, assuming the market demand is a decreasing convex function of buyer’s selling price and a linear increasing function of the supplier’s product quality. In section 5, we report our empirical observations on the impact of transporter’s cost parameters on economic ordering quantities and supply chain profitability under different coordination environment. Finally in section 6, we discuss future research extensions.

2. Models for individual partner’s yearly profitability

Our analysis is based on a supply process with a supplier (a manufacturer or a purchaser), a buyer (a retailer or a distributor who directly faces the market demand), and a third party logistics partner or a transporter who transports the shipment from the supplier to the buyer. We assume that the operation costs, including set up cost per order and the holding cost (per unit per year) incurred to all the partners are known. In addition, we assume the market demand to the product is deterministic, continuous, decreases as buyer’s (retailer’s) market selling price $x$ increases, and increases as supplier’s product quality $q$ improves (or say increases as the supplier’s cost on quality improvement increases). We shall use $D(x,q)$ to denote this continuous price-quality sensitive demand.

Notations used in our analysis are summarized as follows:


\( x \): The buyer’s (or the retailer’s) unit selling price to the market;

\( p \): The buyer’s unit purchasing price \((p < x)\) from the supplier;

\( q \): The product quality level \((0 < q < 1)\), and \( q \Rightarrow 1 \) if the product is of top quality in the respective market;

\( c \): The supplier’s variable cost of manufacturing \((c < p)\);

\( u \): The supplier’s budget on quality for each unit of product, \( u = \varphi(q) \);

\( T(p) \): The transporter’s unit shipping (or mileage) cost, as an increasing function of \( p \);

\( g \): Shipping rate quoted by the transporter, \( g > T(p) \);

\( k \): The transporter’s profit per unit of shipment, as a constant component of \( g \);

\( S_b \): The buyer’s fixed ordering cost per order (e.g., fixed cost per truck);

\( S_s \): The supplier’s fixed processing and set up cost per order;

\( S_t \): The transporter’s fixed cost per order;

\( H_b \): The buyer’s unit holding cost per year;

\( H_s \): The supplier’s unit holding cost per year;

\( H_t \): The transporter’s unit holding cost for inventory-in-transit per year;

\( Q \): The order size (or the shipping quantity) per order;

\( D(x,q) \): The annual market demand, as a function of \( x \) and \( q \).

We assume that the supply process is a free-on-board (FOB)-destination process which requires the supplier to pay for the shipping cost. This implies that the supplier’s selling price \( p \) includes a variable production cost \( c \), a unit product quality cost \( u \) (as one of the decision variables of the model), the shipping rate \( g \), and the profit that the supplier may want to charge from his/her sales to the buyer, or \( c + u + g < p < x \), where \( x \) stands for the buyer (or retailer)’s market selling price. Given the assumptions, the yearly profitability of the supplier, the transporter and the buyer can be represented as

\[
\Pi_s(p,q) = (p-c-u-g)D(x,q) - S_s D(x,q)/Q - H_s Q/2
\]  
\( 1 \)

\[
\Pi_t(g) = (g-T(p))D(x,q) - S_t D(x,q)/Q - H_t Q/2
\]  
\( 2 \)

\[
\Pi_b(x) = (x-p)D(x,q) - S_b D(x,q)/Q - H_b Q/2
\]  
\( 3 \)
As we can see, these yearly profit functions are optimized by different ordering quantities \( Q \). As we can also see, the operation cost of the transporter, defined as a component of \( 2 \), is a concave function of the market demand \( D(x,q) \). Depending on the ordering quantity to be shipped each time, the annual operation cost of the transporter can be represented as

\[
T(p) \cdot D(x,q) + \frac{S_i H_i + S_j H_i}{\sqrt{2S_i H_i}} \sqrt{D(x,q)} \quad i = \text{supplier, buyer or transporter}
\]

which is minimized at \( T(p) \cdot D(x,q) + \sqrt{2S_i H_i} D(x,q) \) when \( i = \) transporter or when the ordering quantity in \( 2 \) is set equal to the economic transportation quantity (ETQ) (see Carter and Ferrin (1995)), or

\[
Q = Q_i = \sqrt{2S_i D(x,q)/H_i}.
\]

When each partner acts for his/her own interest without coordination, it is common in practice to have the following business environment:

**Definition 1.** An independent business environment refers to a decentralized business process where the transporter decides on the shipping rate \( g \). The buyer makes decisions on ordering quantity, or \( Q_b \), that minimizes his/her own yearly fixed ordering and inventory holding cost. The supplier determines selling price \( p \) and quality level \( q \) with a reference to the market benchmarks (Cook and Jackson (2001)) that together maximize supplier’s yearly profit. With any given \( p \) and \( q \), the buyer then determines market selling price \( x \) that maximizes buyer’s yearly profit.

This business environment, if we take the transporter out, has been widely used in a number of previous studies (e.g, see (Weng, 1995)).

In such a business environment, each partner seeks to optimize his/her own profit. In doing so, the buyer’s optimal ordering quantity is

\[
Q_b = \sqrt{2S_b D(x,q)/H_b}
\]

which maximizes buyer’s yearly profit at
\[ \Pi_b(x \mid Q_b) = (x - p)D(x, q) - [2S_bH_bD(x, q)]^{1/2}. \] (3a)

For any given supplier’s price \( p \) and quality level \( q \), the buyer’s interest is to choose his/her optimal market selling price \( x_b^*(p, q) \) that together with

\[ Q_b(p, q) = \sqrt{2S_b D(x_b^*(p, q), q)/H_b} \]

maximize (3a). Given \( x_b^*(p, q) \) and \( Q_b(p, q) \), transporter’s yearly profit becomes

\[ \Pi_t(g) = (g - T(p))D(x_b^*(p, q), q) - (S_t/S_b + H_t/H_b)[S_bH_b D(x_b^*(p, q), q)/2]^{1/2} \] (2a)

and the supplier’s yearly profit becomes

\[ \Pi_s(p, q) = (p - c - u - g)D(x_b^*(p, q), q) - (S_s/S_b + H_s/H_b)[S_bH_b D(x_b^*(p, q), q)/2]^{1/2} \] (1a)

Since the supplier has a complete information about the buyer’s \( Q_b \) and \( x_b^*(p, q) \), he/she can always optimize the values of \( p \) and \( q \) to maximize \( \Pi_s \) upon a given demand function \( D(x, q) \). The maximum supply chain profit that may be possibly achieved under such an independent (decentralized) business environment can now be represented as (see Choi, Lei and Wang, 2002, for details)

\[ \Pi_s + \Pi_t + \Pi_b = (x_b^*(p, q) - c - u - T(p))D(x_b^*(p, q), q) \\
- 2[1 + (S_s + S_t + H_s + H_t)/2][S_bH_b D(x_b^*(p, q), q)/2]^{1/2} \] (4)

Note that (4) remains unchanged regardless the contract between the supplier and the buyer is based on a FOB-destination agreement (the supplier’s expense on shipping cost) or a FOB-origin agreement (the buyer’s expense on shipping cost).

**Example 1.** Consider an independent business environment with \( S_b = 200, S_t = 600, S_c = 4000, H_b = 25, H_s = 20, H_t = 20, \ d = 2 \times 10^{10}, c = 40 \). Suppose \( D(x, q) = dq/x^2 \),
Lemma 1: With the buyer’s economic ordering quantity, \(Q_b(p,q)\), and the transporter’s profit/cost structure (2a), the transporter’s yearly profit is never higher than the maximum (or the transporter’s fixed plus holding cost is never lower than the minimum) that can be achieved by the transporter’s own ETQ, \(Q_i(p,q) = \sqrt{2S_i D(x^*_b(p,q),q)/H_i}\).

Proof: We prove the correctness of the claim by showing that the transporter’s operation (holding plus fixed processing) cost under \(Q_b\) is no less than that under \(Q_i(p,q)\). With \(Q_i(p,q)\), the transporter’s operation cost is \(\sqrt{2S_i H_i D(x^*_b(p,q),q)}\) and with \(Q_b(p,q)\), this costs becomes

\[
\frac{1}{2}[(S_i H_b/S_b H_b)^{1/2} + (S_b H_i/S_i H_i)^{1/2}]\sqrt{2S_i H_i D(x^*_b(p,q),q)}
\]

Since \((S_i H_b/S_b H_b)^{1/2} + (S_b H_i/S_i H_i)^{1/2} \geq 2(S_i H_b/S_b H_b)^{1/4} (S_i H_i/S_i H_i)^{1/4} = 2\), the claim holds.

Lemma 2: With the buyer’s economic ordering quantity \(Q_b(p,q)\), the supplier’s yearly profit is never higher than the maximum (or the supplier’s fixed plus holding cost is
never lower than the minimum) that can be achieved by the supplier’s own EOQ:

\[ Q_s(p,q) = \sqrt{2S_s D(x_s^*(p,q)) / H_s}. \]

**Proof:** (Similar to the proof to Lemma 1 and is thus skipped).

Note that result stated in Lemma 2 is consistent with the observation made by Kevin Weng (1995) in his study on the buyer-supplier (without transporter) models. Our study reconfirms that, even when a third party logistics partner with a non-negligible operation cost is involved, this known observation continues to hold.

In many supply chain processes, vendor managed inventory (VMI) has been shown to be an effective strategy for reducing the cost and improving the performance (see the work by Aviv and Federgruen (1998) and Cheung and Lee (2002)). In a VMI environment, it is the supplier that makes decisions on the timing and the quantity \( Q_s \) to be delivered to the buyer. When the supplier makes his/her ordering quantity decision with \( Q_s = (2S_s D(x,q) / H_s)^{1/2} \), the transporter’s shipping quantity changes to \( Q_s \), and the buyer’s yearly profit becomes

\[
\Pi_b(x \mid Q_s) = (x - p)D(x,q) - (S_h / S_s + H_b / H_s)[S_s H_s D(x,q) / 2]^{1/2}.
\]

In this case, we can also show that the following results hold (proofs skipped).

**Lemma 3:** With the supplier’s \( Q_s(p,q) \), the buyer’s yearly profit is never higher than the maximum that can be achieved under the buyer’s \( Q_b(p,q) = \sqrt{2S_b D(x_s^*(p,q),q) / H_b} \).

**Lemma 4:** With the supplier’s \( Q_s(p,q) \) and the transporter’s profit/cost structure (2), the transporter’s yearly profit is never higher than the maximum that can be achieved by the transporter’s own ETQ: \( Q_t(p,q) = \sqrt{2S_t D(x_s^*(p,q),q) / H_t} \).
The discussion above is sufficient to lead to a conclusion that, in an independent business environment, each individual partner makes a maximum yearly profit that is no more than that can be achieved by his/her own economic ordering quantity, regardless the supply chain ordering quantity is decided by the buyer or the supplier. This conclusion also implies that the sum of maximum yearly profit achieved by individual partners in an independent business environment, \( \Pi_s + \Pi_t + \Pi_b \), is a lower bound of that may possibly be achieved by a centralized channel coordination, as we will discuss in Section 3.

As a motivation to our analysis on centralized channel coordination in next section, the following result shows that the transporter’s yearly profitability (2a) may be improved even in an independent/decentralized business environment if the buyer incorporates, in his/her decisions on ordering quantity, the operation costs incurred to all the other partners in the supply chain. Let \( S_J = S_s + S_b + S_t, \quad H_J = H_s + H_b + H_t \), and \( Q_J = \sqrt{2S_JD(x,q)/H_J} \).

**Lemma 5**: In an independent business environment, for any given \( p, q, g \) and \( x \), if

\[
\frac{S_s + S_t}{S_b} \geq \frac{H_s + H_t}{H_b} \quad \text{and} \quad \frac{S_t}{H_t} \geq \sqrt{\frac{S_b}{H_b} \cdot \frac{S_J}{H_J}},
\]

then \( \Pi_t(g \mid Q_J) \geq \Pi_t(g \mid Q_b) \), where

\[
\Pi_t(g \mid Q_J) = (g - T(p))D(x,q) - \left( \frac{S_t}{S_j} + \frac{H_t}{H_J} \right)[H_JD(x,q)/2]^{1/2},
\]

and

\[
\Pi_t(g \mid Q_b) = (g - T(p))D(x,q) - \left( \frac{S_t}{S_b} + \frac{H_t}{H_b} \right)[H_bD(x,q)/2]^{1/2}.
\]

**Proof**: The claim \( \Pi_t(g \mid Q_J) \geq \Pi_t(g \mid Q_b) \) holds if and only if

\[
\frac{(S_t + H_t)^2}{S_b} S_b H_b - \frac{(S_t + H_t)^2}{H_J} S_J H_J \geq 0.
\]

Since

\[
\frac{(S_t + H_t)^2}{S_b} S_b H_b - \frac{(S_t + H_t)^2}{H_J} S_J H_J = \left[ \frac{S_t^2}{S_b} \frac{H_b}{H_J} - \frac{S_t}{H_J} \frac{S_b}{S_J} \right] \quad \text{and}
\]

\[
= S_t^2 \left[ \frac{H_b}{S_b} \frac{H_J}{S_J} \right] - H_t^2 \left[ \frac{S_J}{H_J} \frac{S_b}{H_b} \right].
\]
\[
\frac{S_s + S_t}{S_b} \geq \frac{H_s + H_t}{H_b}, \quad \text{and} \quad \frac{S_t}{H_t} \geq \sqrt[2]{\frac{S_b}{H_b} \cdot \frac{S_J}{H_J}}, \quad \text{we have}
\]

\[
S_t^2 \left[ \frac{H_b}{S_b} - \frac{H_J}{S_J} \right] - H_t^2 \left[ \frac{S_J}{H_J} - \frac{S_b}{H_b} \right]
\]

\[
= \left[ \frac{S_t^2}{S_b S_J} - \frac{H_t^2}{H_b H_J} \right] \left[ H_b (S_s + S_t) - (H_s + H_t)S_b \right] \geq 0.
\]

Among the two conditions, \(\frac{S_s + S_t}{S_b} \geq \frac{H_s + H_t}{H_b}\) and \(\frac{S_t}{H_t} \geq \sqrt[2]{\frac{S_b}{H_b} \cdot \frac{S_J}{H_J}}\), we see that \(\frac{S_s + S_t}{S_b} \geq \frac{H_s + H_t}{H_b}\) can be easily justified since \(S_t + S_s \gg S_b, H_t \approx H_s \approx H_b\) hold for most applications. However, condition \(\frac{S_t}{H_t} \geq \sqrt[2]{\frac{S_b}{H_b} \cdot \frac{S_J}{H_J}}\) holds only for the situations where the fixed cost of the transporter is relative higher than the fixed cost of the buyer.

In particular, we can show that the condition \(\frac{S_t}{H_t} \geq \sqrt[2]{\frac{S_b}{H_b} \cdot \frac{S_J}{H_J}}\) holds whenever \(S_b \leq \frac{1}{m + 1} S_t\), and \(S_t \geq \frac{1}{m} (S_b + S_s)\), where parameter \(m \geq 1\) can be any positive integer.

Figure 1 below, as an example, shows the improvement on transporter’s yearly profit when the buyer’s ordering quantity changes from \(Q_b\) to \(Q_J\). In this example, the transporter’s yearly profitability increased by \([(0.65-0.48)/0.48] \%\) or 35\% after the buyer changes his/her ordering quantity from \(Q_b\) to \(Q_J\). In practice, this change in the buyer’s decision, if can be implemented, could offer a strong incentive to the transporter which in turn may lead to a better coordination and better logistics services.
3. Models for channel coordination

We now investigate the potential improvement on supply chain profitability by a centralized coordination or a total supply chain coordination where all the three partners are willing to make joint decisions to maximize (and then share) the total profitability of the supply chain as a whole entity.

In a total coordination environment, the joint yearly profit of the supplier, the buyer, and the transporter becomes

\[
\Pi_j (x, p, q, Q) = \Pi_s + \Pi_t + \Pi_b
\]

\[
= (x - c - u - T(p))D(x, q) - (S_s + S_b + S_t)D(x, q)/Q - (H_s + H_b + H_t)Q/2
\]

\[
= (x - c - u - T(p))D(x, q) - S_j \frac{D(x, q)}{Q} - H_j \frac{Q}{2}
\]  

(5)

where \( S_j = S_s + S_b + S_t, \ H_j = H_s + H_b + H_t \)

The optimal ordering quantity that minimizes the joint operation costs (holding plus fixed ordering) of all the three partners is
which maximizes the total supply chain profitability (5) at

\begin{equation}
\Pi_j(x, p, q | Q_j) = (x - c - u - T(p))D(x, q) - [2S_jH_jD(x, q)]^{1/2}
\end{equation}

(5a)

for any given supplier’s \( p, q, \) transporter’s \( g, \) and buyer’s price \( x. \)

**Theorem 1.** (Dominance relationship) For any given \( p, q, g \) and \( x, \) the total supply chain profitability under \( Q_j \) dominates the sum of individual partners’ maximum yearly profit in an independent business environment using ordering quantity \( Q_b. \) That is

\[ \Pi_j(x, p, q | Q_j) \geq \Pi_s + \Pi_t + \Pi_b. \]

**Proof:** From (1a), (2a) and (3a), we have

\[
\Pi_s + \Pi_t + \Pi_b = (x - c - \varphi(q) - T(p))D(x, q) - 2[1 + \left( \frac{S_s + S_t}{S_b} + \frac{H_s + H_t}{H_b} \right)/2][S_bH_bD(x, q)/2]^{1/2}
\]

Since

\[
\Pi_j(x, p, q | Q_j) = (x - c - \varphi(q) - T(p))D(x, q) - 2[1 + \frac{S_s + S_t}{S_b} + \frac{(S_s + S_t)(H_s + H_t)}{S_bH_b} + \frac{H_s + H_t}{H_b}]^{1/2}[S_bH_bD(x, q)/2]^{1/2}
\]

and

\[
[1 + \frac{S_s + S_t}{S_b} + \frac{H_s + H_t}{H_b}/2] - [1 + \frac{S_s + S_t}{S_b} + \frac{(S_s + S_t)(H_s + H_t)}{S_bH_b} + \frac{H_s + H_t}{H_b}] \geq 0,
\]

the claim holds. 

In a very similar way, we can prove the correctness of the following result.

**Theorem 2:** (Generalized dominance relationship) For any given \( p, q, g \) and \( x, \) we have

\[ \Pi_j(x, p, q | Q_j) \geq \Pi_s + \Pi_t + \Pi_b \]

regardless the ordering quantity \( Q \) for the independent business environment is decided by which partner, or say regardless \( Q = Q_s, Q_t, \) or \( Q_j. \)
**Theorem 3:** The total supply chain profitability achieved under joint ordering quantity $Q_j$, $\Pi_j^*(p)$, increases as the supplier’s unit selling price $p$ decreases.

**Proof:** Assume that $p_1 < p_2$, then

\[
\Pi_j^*(p_1) = \Pi_j^*(x_j^*, q_j^* | p_1) = (x_j^*(p_1) - c - u_j^*(p_1) - T(p_1))D(x_j^*(p_1), q_j^*(p_1)) - [2S_j H_j D(x_j^*(p_1), q_j^*(p_1))]^{1/2}
\]

\[
\Pi_j^*(p_2) = \Pi_j^*(x_j^*, q_j^* | p_2) = (x_j^*(p_2) - c - u_j^*(p_2) - T(p_2))D(x_j^*(p_2), q_j^*(p_2)) - [2S_j H_j D(x_j^*(p_2), q_j^*(p_2))]^{1/2}
\]

\[
\Pi_j^*(p_1) \geq (x_j^*(p_1) - c - u_j^*(p_1) - T(p_1))D(x_j^*(p_1), q_j^*(p_1)) - [2S_j H_j D(x_j^*(p_1), q_j^*(p_1))]^{1/2}
\]

In addition, we have

\[
\Pi_j^*(p_1) = \Pi_j^*(p_2)
\]

Theorem 3 reveals the importance of supplier’s coordination. If the supplier is willing not to charge a profit from his/her sales to the buyer, then the total supply chain profitability will be higher than otherwise they may achieve. This observation also leads to the following result.

**Corollary 1:** In a joint channel coordination environment, for any given $g$, the supplier’s optimal unit selling price $p_j^*$ are given by $\Pi_j(p_j^*) = 0$ or

\[
p_j^* = c + u_j^* + g + S_j / Q_j + H_j Q_j / [2D(x_j^*, q_j^*)]
\]

In addition, we have

**Theorem 4:** The total supply chain profitability achieved under joint ordering quantity $Q_j$, $\Pi_j^*(g)$, increases as the transporter’s shipping rate $g$ decreases.

**Proof:** Assume that $g_1 < g_2$, then
Theorem 4 reveals the importance of transporter’s coordination. If the transporter is willing not to charge a profit from his/her contract with the supplier (in a FOB-destination agreement), then the resulting total supply chain profitability will be higher than otherwise they may achieve. This observation then leads to the following result.

**Corollary 2**: In a joint channel coordination environment, the transporter’s optimal shipping rate charged to the supplier, \( g^*_j \) is given by \( \Pi_i(g^*_j) = 0 \) or

\[
g^*_j = T(p^*_j) + S_i/Q_j + H_i Q_j/[2D(x^*_j, q^*_j)], \text{ with } k^*_j = 0.
\]

As a conclusion to the discussion above, we have the following result (with proof skipped).

**Theorem 5**: In a joint channel coordination environment, the supplier’s optimal unit selling price \( p^*_j \) and the transporter’s optimal unit transportation price charged to the supplier, \( g^*_j \) are given by

\[
\Pi_j^*(g_1) = \Pi_j^*(x^*_j, q^*_j, p^*_j | g_1)
\]

\[
= [x^*_j(g_1) - c - u^*_j(g_1) - T[c + u^*_j(g_1) + g_1 + (S_i/S_j + H_i/H_j)(S_j H_j D(x^*_j, q^*_j(g_1))/2)]^{1/2}
\]

\[
D(x^*_j(g_1), q^*_j(g_1)) - [2S_j H_j D(x^*_j, q^*_j(g_1))]^{1/2}
\]

\[
\Pi_j^*(g_2) = \Pi_j^*(x^*_j, q^*_j, p^*_j | g_2)
\]

\[
= [x^*_j(g_2) - c - u^*_j(g_2) - T[c + u^*_j(g_2) + g_2 + (S_i/S_j + H_i/H_j)(S_j H_j D(x^*_j, q^*_j(g_2))/2)]^{1/2}
\]

\[
D(x^*_j(g_2), q^*_j(g_2)) - [2S_j H_j D(x^*_j, q^*_j(g_2))]^{1/2}
\]

\[
\Pi_j^*(g_1) \geq [x^*_j(g_2) - c - u^*_j(g_2) - T[c + u^*_j(g_2) + g_2 + (S_i/S_j + H_i/H_j)(S_j H_j D(x^*_j, q^*_j(g_2))/2)]^{1/2}
\]

\[
D(x^*_j(g_2), q^*_j(g_2)) - [2S_j H_j D(x^*_j, q^*_j(g_2))]^{1/2}
\]

\[
> [x^*_j(g_2) - c - u^*_j(g_2) - T[c + u^*_j(g_2) + g_2 + (S_i/S_j + H_i/H_j)(S_j H_j D(x^*_j, q^*_j(g_2))/2)]^{1/2}
\]

\[
D(x^*_j(g_2), q^*_j(g_2)) - [2S_j H_j D(x^*_j, q^*_j(g_2))]^{1/2} = \Pi_j^*(g_2)
\]
\[
\begin{align*}
    p_j^* &= c + u_j^* + g_j^* + S_j/Q_j + H_jQ_j/[2D(x_j^*, q_j^*)] \\
    g_j^* &= T(p_j^*) + S_j/Q_j + H_jQ_j/[2D(x_j^*, q_j^*)], \text{ with } k_j^* = 0
\end{align*}
\] (7)

In next section, we shall focus our analysis on the optimal operation policies when the market demand is a continuous decreasing convex function of buyer’s selling price \(x\) and a linear increasing function of supplier’s product quality \(q\), and when the shipping rate \(g\) is a continuous function of demand \(D(x,q)\).

4. The impact of channel coordination under \(D(x,q) = \frac{dq}{x^2}\)

In this section, we develop optimal operation policies, in terms of selling prices, product quality and ordering quantity, that together maximize the total profitability of a supply chain, assuming that the market demand has a structure of \(D(x,q) = dq/x^2\), where parameter \(d \gg 1\) stands for the market factor. Furthermore, we assume that the supplier’s product quality \(q\) can be modeled as a continuous increasing function of \(u\), \(q = e^{-u}, 0 < q < 1\), where \(u\) stands for the dollar amount the supplier invests to ensure the quality of each unit of product. We assume the transporter’s shipping rate \(g\) to be

\[
g = k + T(p) + S/Q + H/Q/[2D(x,q)], \quad T(p) = ap
\]

or

\[
g = k + ap + \left[\frac{S_H + H_S}{\sqrt{2SHD(x,q)}}\right]
\]

where \(k\) is a constant representing transporter’s profit charge, \(ap\) stands for the unit shipping operation cost, and \(S\) and \(H\) are dependent of the ordering quantity \(Q\).

4.1. Optimal policies under a decentralized environment
In an independent/decentralized business environment, the buyer applies and executes an order quantity that maximizes his/her own profitability for any given $p$ and $q$. That is $Q_b = \sqrt{2S_b D(x, q)/H_b}$ and
\[
\Pi_b(x) = (x - p)D(x, q) - S_b D(x, q)/Q - H_b Q/2
\]
\[
= (x - p)\frac{dq}{x^2} - \sqrt{2S_b H_b \frac{dq}{x^2}}
\]
\[
= \frac{d}{x} e^{-\frac{u}{x}} - \frac{dp}{x^2} e^{-\frac{u}{x}} - \frac{[2dS_b H_b]}{x} e^{-\frac{1}{2u}}
\]

Then, the following results hold.

**Theorem 6.** For an independent business environment where the market demand is approximated by $D(x, q) = \frac{dq}{x^2}$, the buyer’s optimal selling price that maximizes his/her yearly profitability $\Pi_b$ is

\[
x_b^* = \frac{2}{1 - \frac{2S_b H_b}{dq}}, p
\]

for any given supplier’s selling price $p$ and product quality level $q$.

**Proof.** Let $\frac{d\Pi_b}{dx} = -\frac{d}{x^2} e^{-\frac{u}{x}} + \frac{2dp}{x^2} e^{-\frac{u}{x}} - \frac{[2dS_b H_b]}{x} e^{-\frac{1}{2u}}$, we have

\[
x_b^* = \frac{2dpe^{-\frac{u}{x}}}{de^{-\frac{u}{x}} - \frac{[2dS_b H_b]}{x} e^{-\frac{1}{2u}}} = \frac{2p}{1 - \frac{2S_b H_b}{dq}} = x_b^* = \frac{2}{1 - \frac{2S_b H_b}{dq}} p
\]

From the analysis above, we can also see that when the supplier has a top quality in the market (i.e., $q \Rightarrow 1$) and when the supply chain’s market share is sufficiently high (i.e., $d \gg S_b H_b$), then the buyer’s optimal selling price can be approximated by
for any given supplier’s price \( p \).

**Theorem 7.** In an independent business environment, if the market demand can be approximated by \( D(x, q) = \frac{dq}{x^2} \) and if the supplier’s unit production cost \( c \) is significantly greater than 1, then the supplier’s optimal investment into product quality \( u^*_b \) should be close enough to \( u^*_b \approx \sqrt{c + k} \) and the optimal quality level that the supplier should attain is \( q^*_b = e^{\frac{1}{\sqrt{c + k}}} \), where parameter \( k \) stands for the profit that transporter charges for the shipment of each unit of product.

**Proof.** Given \( x^*_b \) defined by (8a), the supplier’s yearly profit becomes

\[
\Pi_s = (p - c - u - g)D(x, q) - S_x D(x, q)/Q - H_s Q/2
\]

\[
= (p - c - u - k - ap)D(x^*_b, q) - [S_x + S_t]/S_b + (H_s + H_t)/H_b][S_b H_b D(x^*_b, q)/2]^{1/2}
\]

\[
= (p - c - u - k - ap)[d^{1/2}e^{-\frac{1}{2u}} - (2S_b H_b)^{1/2}]^2
\]

\[
4p^2
\]

\[-[(S_x + S_t)/S_b + (H_s + H_t)/H_b][S_b H_b /2]^{1/2} d^{1/2}e^{-\frac{1}{2u}} - (2S_b H_b)^{1/2}
\]

\[
2p
\]

Let \( \frac{\partial \Pi_s}{\partial p} = 0 \) and solve for \( p \), we have

\[
\frac{\partial \Pi_s}{\partial p} = -(1 - a)[d^{1/2}e^{-\frac{1}{2u}} - (2S_b H_b)^{1/2}]^2
\]

\[
+ (c + u + k)[d^{1/2}e^{-\frac{1}{2u}} - (2S_b H_b)^{1/2}]^2 \]

\[
2p^3
\]

\[
+ [(S_x + S_t)/S_b + (H_s + H_t)/H_b][S_b H_b /2]^{1/2} d^{1/2}e^{-\frac{1}{2u}} - (2S_b H_b)^{1/2}
\]

\[
2p^3
\]

\[= 0
\]
\[ p = \frac{2(c + u + k)(d^{\frac{1}{2}}e^{-\frac{1}{2u}} - (2S_bH_b)^{\frac{1}{2}})}{(1 - a)(d^{\frac{1}{2}}e^{-\frac{1}{2u}} - (2S_bH_b)^{\frac{1}{2}}) - [(S_s + S_r)/S_b + (H_s + H_t)/H_b][2S_bH_b]^{\frac{1}{2}}} \]

\[ = \frac{2(c + u + k)(d^{\frac{1}{2}}e^{-\frac{1}{2u}} - (2S_bH_b)^{\frac{1}{2}})}{(1 - a)d^{\frac{1}{2}}e^{-\frac{1}{2u}} - [1 - a + (S_s + S_r)/S_b + (H_s + H_t)/H_b][2S_bH_b]^{\frac{1}{2}}} \]

Now let \( \frac{\partial \Pi_s}{\partial u} = 0 \) and then solve for \( p \) again

\[ \frac{\partial \Pi_s}{\partial u} = -\left[ \frac{d^{\frac{1}{2}}e^{-\frac{1}{2u}} - (2S_bH_b)^{\frac{1}{2}}}{4p^2} \right]^2 + (p - c - u - k - ap)\left( \frac{d^{\frac{1}{2}}e^{-\frac{1}{2u}} - (2S_bH_b)^{\frac{1}{2}}}{4p^2u^2} \right) d^{\frac{1}{2}}e^{-\frac{1}{2u}} \]

\[ -[(S_s + S_r)/S_b + (H_s + H_t)/H_b][S_bH_b/2]^{\frac{1}{2}} \frac{d^{\frac{1}{2}}e^{-\frac{1}{2u}}}{4pu^2} = 0 \]

this gives

\[ p = \frac{2(c + u + k)(d^{\frac{1}{2}}e^{-\frac{1}{2u}} - (2dS_bH_b)^{\frac{1}{2}}) + 2[d^{\frac{1}{2}}e^{-\frac{1}{2u}} - (2S_bH_b)^{\frac{1}{2}}]^2u^2e^{\frac{1}{2u}}}{2(1 - a)(d^{\frac{1}{2}}e^{-\frac{1}{2u}} - (2dS_bH_b)^{\frac{1}{2}}) - [(S_s + S_r)/S_b + (H_s + H_t)/H_b][2dS_bH_b]^{\frac{1}{2}}} \]

Equations (9) and (10) together define the following relationship

\[ (1 - a)[u^2 - u - (c + k)]e^{-\frac{1}{2u}} - u^2 [1 - a + (S_s + S_r)/S_b + (H_s + H_t)/H_b][2S_bH_b/d]^{\frac{1}{2}} = 0 \]

Since \( d >> S_bH_b \), the relationship above can be approximated by

\[ u^2 - u - (c + k) = 0 \]

or

\[ u_b^* = \frac{1 + \sqrt{1 + 4(c + k)}}{2} \]  \( \text{(11)} \)

Equation (11) implies that when \( c >> 1 \), \( u_b^* \approx \sqrt{c + k} \), and \( q_b^* = e^{-\frac{1}{\sqrt{c+k}}} \). \( \blacksquare \)

Results (8a), (8b), and (11) provide guidelines for the optimal operational policies that the buyer and the supplier should follow, for a given shipping rate \( g_s \), to maximize their own yearly profitability in an independent/decentralized business environment. In
next section, we shall switch our analysis to the optimal operational policies in a totally coordinated environment.

4.2 Optimal policies under a centralized (total coordinated) environment

When all the partners (the supplier, the buyer, and the transporter) are willing to coordinate their operations to optimize the total supply chain profitability, the optimal joint ordering quantity becomes $Q_j = (2S_j D(x,q)/H_j)^{1/2}$ which maximizes the supply chain yearly profitability at

$$
\Pi_j = (x - c - u - ap) \frac{dq}{x^2} - \sqrt{2S_j H_j \frac{dq}{x^2}}
$$

$$
= \frac{d}{x} e^{-u} - (c + u + ap) \frac{d}{x^2} e^{-u} \left[ \frac{2dS_j H_j}{x} \right] \frac{1}{e^{-2u}}
$$

Let $\frac{\partial \Pi_j}{\partial x} = -\frac{d}{x^2} e^{-u} + (c + u + ap) \frac{2d}{x^3} e^{-u} \left[ \frac{2dS_j H_j}{x} \right] \frac{1}{e^{-2u}} = 0$, we obtain

$$
x_j^* = \frac{2(c + u + ap) \frac{1}{e^{-u}}}{\frac{1}{de^{-u}} - \left[ \frac{2dS_j H_j}{x} \right] \frac{1}{e^{-2u}}} = \frac{2(c + u + ap)}{1 - \frac{2S_j H_j}{dq}}
$$

This leads to the following result (proof skipped).

**Theorem 8.** If the market demand can be approximated by $D(x,q) = \frac{dq}{x^2}$, then the optimal market selling price that maximizes the joint supply chain profitability (12) is defined by

$$
x_j^* = \frac{2(c + u + ap)}{1 - \sqrt{\frac{2S_j H_j}{dq}}}
$$

for any given supplier’s decision on product quality $u$ and $q$. 
Furthermore, let
\[
\frac{\partial \Pi_J}{\partial u} = -\frac{d}{x}\frac{1}{e^{\frac{1}{u}}} + (x - c - u - ap)\frac{d}{x^2u}e^{-\frac{1}{u}} - \frac{1}{2} \frac{(2dS_JH_J)'}{2xu^2}e^{-\frac{1}{2u}} = 0
\]

We obtain
\[
x = \frac{2(c + u + ap)de^{-\frac{1}{u}} + 2du^2e^{-\frac{1}{u}}}{2de^{-\frac{1}{u}} - [2dS_JH_J]'e^{-\frac{1}{2u}}} \tag{14}
\]

From (13) and (14), the following relationship holds
\[
[u^2 - u - (c + ap)]e^{-\frac{1}{2u}} - u^2(2S_JH_J/d)^{1/2} = 0
\]

Since \( d >> S_JH_J \), the relationship above reduces to \( u^2 - u - (c + ap) = 0 \) or
\[
u^*_J = \frac{1 + \sqrt{1 + 4(c + ap)}}{2} \tag{15}
\]

We now have the following conclusion.

**Theorem 9.** In a total coordination environment, if the market demand can be approximated by \( D(x,q) = \frac{dq}{x^2} \) and if the supplier’s unit production cost \( c \) is significantly greater than 1, then the supplier’s optimal investment into product quality improvement \( u^*_J \) should be close enough to \( u^*_J \approx \sqrt{c + ap} \) and the optimal quality level that the supplier should attain is \( q^*_J = e^{-\frac{1}{\sqrt{c+ap}}} \).

**Proof.** (Derivable from (15) and is thus skipped).

Given \( D(x,q) = \frac{dq}{x^2} \), one can also verify that the supplier’s optimal selling price \( p^*_J \) and the transporter’s optimal shipping rate \( g^*_J \), that together with \( x^*_J \) and \( q^*_J \), maximize the joint yearly profit of a supply chain are determined by
\[
\begin{align*}
    p_j^* &= c + u_j^* + g_j^* + (S_j / S_j + H_j / H_j) (S_j H_j x_{j}^{*2} / (2dq_j^{*}))^{1/2}, \\
    g_j^* &= ap_j^* + (S_j / S_j + H_j / H_j) (S_j H_j x_{j}^{*2} / (2dq_j^{*}))^{1/2}, \text{ with } k_j = 0
\end{align*}
\] (16)

5. Empirical results

To help our understanding on the impact of transporter’s operation costs and channel coordination policies on supply chain profitability, we have also conducted a series of empirical studies. Figures 2 to 8 present our observations from these studies. Parameters used in the empirical studies are summarized as follows.

\[d = 2 \times 10^10, \ 10\% \leq S_j / S_j \leq 55\%, \quad 0 \leq k / c \leq 45\%, \quad 0 < a \leq 10\%, \quad D(x,q) = dq / x^2\]

Figure 2 shows the relationships between the economic ordering quantities and the transporter’s unit shipping costs, \(ap\), with respect to different levels of coordination (i.e., an independent business environment and a totally coordinated environment, respectively). As we can see, the economic ordering quantities decrease as the unit shipping cost \(ap\) increases. This is due to the fact that \(c + u + ap + k < p < x\). As \(ap\) increases, supplier’s selling price \(p\) and then buyer’s market selling price \(x\) will be forced to increase monotonically, which in turn make the market demand \(D(x,q) = dq / x^2\) and the ordering quantity \(Q = \sqrt{2SD(x,q)} / H\) to decrease. This decrease in ordering quantity is more significant under a total coordination environment than that under an independent business environment. Another interesting observation is revealed by the results summarized in Figure 3. As transporter’s fixed cost, \(S_j\), increases, the economic ordering quantity in an independent business environment (i.e., buyer-based) remains essentially the same since \(Q_0\) is not affected by \(S_j\). However, the optimal ordering quantity in a totally coordinated environment will increase quickly and tends to find the optimal balance between the total holding and the total fixed ordering cost incurred along the supply process whenever an order is placed. This empirical observation reveals the
sub-optimality of the classical economic ordering quantity for the two-partner (supplier-buyer), $Q_s$, when applied to a supply chain process where the transporter’s operation cost is non-negligible.

Figures 4, 5 and 6 report our observations on the relationship between supply chain profitability and operation costs of the transporter. Figure 4 shows the impact of transporter’s unit shipping cost and Figure 5 shows the impact of transporter’s fixed cost. As we can see from Figure 5, the gap between the supply chain profitability achieved in different business environment increases significantly and quickly as the transporter’s fixed shipping cost increases. This also shows that the total profitability in an independent business environment is more vulnerable to the changes in supply chain operation costs.

Figure 6 shows the impact of the transporter’s profit charge, $k$, on the total profitability under different business environment. As the ratio $k/c$, where parameter $c$ stands for the unit variable manufacturing cost of the supplier, increases, the supply chain profitability in a total coordinated environment remain to be the same because $k=0$ is required by optimal operation policies. However, the supply chain profitability achieved
under an independent business environment decreases very quickly. As the ratio $k/c$ approaches to 45%, the total supply chain profitability dropped by 28.5% in an independent business environment.
Figure 7 shows how the supply chain profitability is affected by the product quality $q$, where $0 < q < 1$. From (11) and (15), the product quality levels that maximize the total profitability are $q^* \approx e^{-\frac{1}{\sqrt{c+k}}}$ and $q^* \approx e^{-\frac{1}{\sqrt{c+ap}}}$, in an independent, and in a totally coordinated, environment, respectively. As we may see, if the market demand can be approximately modeled as $D(x,q) = dq / x^2$, then pushing a quality level $q \Rightarrow 1$ may not necessarily maximize the supply chain profitability, neither for an independent business environment nor for a totally coordinated environment. As actual quality level $q$ approaches $q^*$, the total profitability increases, and as $q$ goes beyond $q^*$, the total profitability will taper down quickly due to manufacturer’s over spending on quality cost $u$. Finally, Figure 8 shows another interesting observation, as supplier’s selling price to the buyer (relatively measured by $p/c$) increases, the total profitability, especially the total profitability achieved under an independent business environment, decreases significantly. This empirical observation is consistent with the conclusion of Theorem 5 in Section 3. That is, the maximum total profitability occurs when none of the intermediate partner charges a profit from his/her down stream partner, except the buyer (the retailer) charges from the market.

$S_b=200/$order, $S_s=4,000/$order, $S_t=600/$order, $H_b=25/(unit, year)$, $H_s=20/(unit, year)$, $H_t=20/(unit, year)$, $c=40/unit$, $k=8/unit$, $a=0.05$
6. Conclusion and future extensions

We studied the impact of partner coordination on the total profitability of a supply chain involving a buyer (the retailer), a supplier, and a transporter with a non-negligible concave shipping cost. The market demand is assumed to be sensitive to both retailer’s (buyer’s) selling price and supplier’s product quality. We have shown that the total supply chain profitability achieved under the ordering quantity jointly determined by the buyer, the transporter, and the supplier dominates the sum of the maximum profitability of individual partners regardless the ordering quantity is determined by the buyer, the transporter, or the supplier. We have also proposed optimal selling prices, quality level and ordering quantity for both an independent business environment and a totally coordinated environment, respectively, assuming the market demand can be modeled as $D(x,q) = \frac{dq}{x^3}$. Empirical studies that further reveal the impact of channel coordination on total supply chain profitability were reported.

This work can be extended in several directions. First, we have not considered any mechanism that may be applied to ensure an effective implementation and the delivery of the promises of the optimal operation policies. Such mechanisms should include those that make the supplier not to charge a profit to the buyer, and the transporter not to charge a profit to the supplier. Such mechanisms should also include those that make the buyer to place the joint economic ordering quantity instead of his/her own economic ordering quantity. There is a significant amount of work remains to be done to realize the practical value of the theoretical results. In addition, such mechanisms should include those that ensure the partners to share the cost structure information. Without an effective mechanism for the implementation of optimal operation policies, the actual improvement on supply chain profitability even when changing from an independent business environment to a totally coordinated environment could be far less significant than that discussed in this paper. Second, it is important to investigate the impact of uncertainty on the optimality of operational policies proposed in this study. We have assumed a known and deterministic market demand, which may not be the case in
practice, especially for retail industries where demand could be highly seasonal, random, and subject to other factors as well. We have also assumed the availability of cost structures of all partners involved in a supply chain process, which however may not always be the case either. Third, we have only considered one type of demand function $D(x,q) = dq / x^2$ in the analysis. The actual demand pattern in practice may deviate tremendously depending on the product and the market. A further analysis to evaluate the robustness of these optimal policies will be interesting.

References:


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