Coordinating a Three-Partners Supply Chain via Quantity Discount

September, 2004

S.Chan Choi*, **Lei Lei***, **and Qiang Wang****

* Authors are listed alphabetically. Rutgers Business School, Rutgers University, 180 University Ave., Newark, NJ 07102

** School of International Trade and Economics, University of International Business and Economics, Beijing, 100029, P.R. of China

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Abstract

Coordinating activities among members in a supply chain has been a subject of great attention in the past two decades. Among various methods of coordination, quantity discount has been suggested in a number of studies as a mechanism to achieve incentive-compatible coordination between a supplier and his buyer, which does not require coercive measures to enforce cooperative behavior of a channel partner. Two separate streams of quantity discount models have been developed in the literature. The operations management literature views quantity discount as a way to minimize the system-wide cost of operation. The marketing literature employs quantity discount to induce the buyer to lower the retail price in order to increase demand. In both models, the supply chain members' profit goals are aligned with the total system's profit, and the resulting coordination creates efficiency gains that can be shared between the members. This paper presents a model of quantity discount that combines these two sources of efficiency gains. We briefly review the existing approaches, and then develop a combined model of quantity discount that also includes a third-party logistics partner. In addition, we examine the role of quality as part of the coordination decisions. We show that coordinating the three-partner supply chain can be as easily implemented as the traditional two-partner model.

1. Introduction

Coordinating activities across multiple players in a supply chain has been a subject of numerous studies in operations management and marketing. A system-level optimal solution that integrates the participants' activities requires that all involved channel members implement respective actions as prescribed by the solution. This requirement would be difficult to enforce unless (i) there is a binding agreement among the channel members, (ii) the actions are forced and policed, or (iii) the prescribed solution is *incentive compatible*. In the last mechanism, the behavior of a participant who acts on self-interest motivation is aligned with the objectives of the other members in the supply channel. It is a more attractive mechanism of channel coordination that benefits the whole system, and it does not require the buyer's coerced "cooperation."

Quantity discount has been proposed in two separate research streams as a tool for achieving incentive compatible coordination within the chain (Dolan 1987). Suppose the supply chain consists of two members: the manufacturer (or the supplier) who determines the wholesale price and the retailer (or the buyer) who chooses his optimal order quantity *and* the retail price. In the operations management literature, the channel's total transaction cost that includes inventory holding, facility set-up, and order processing costs can be minimized by properly designing the quantity discount schedule so that the buyer orders the channel-optimal economic order quantity. This is called the *transaction efficiency*. In the marketing literature, on the other hand, the supplier can offer the buyer a quantity discount, which induces the buyer to choose her price at the joint-optimal level. This eliminates double marginalization, and the increased market demand due to the lower retail price benefits the whole system. This is called the *channel efficiency*.

A few recent studies attempted to combine these two research streams to provide a more comprehensive quantity discount models that combine the two types of efficiencies (Weng 1995; Chen, Federgruen and Zheng 2001). While all these studies focus on a two-member supply channel system (i.e., a buyer and a seller), the model's applicability can be enhanced by extending to the case in which there are more than two supply chain partners. In this paper, we present a quantity discount mechanism for coordinating a supply chain that also includes a third type of member—a third-party transportation partner. We show that the combined efficiencies can also be achieved from coordinating the two different types of down-stream supply chain partners by providing a proper quantity discount schedule. In addition, we also extend the model by including the supplier's decision of choosing optimal quality levels as part of supply chain coordination.

In the next section, we briefly review the existing quantity discount models for supply chain coordination. Section 3 develops the three-partners model and discusses its properties via numerical examples. The last section concludes the paper by delineating future research topics.

2. Quantity Discount Models for Supply Chain Coordination

Channel Efficiency Models

These models have evolved from the marketing problem of channel coordination (Jeuland and Shugan 1983; McGuire and Staelin 1986). The basic assumption is that the demand of a product is a decreasing function of its retail price. The supplier's decision variable is his wholesale price, which becomes the variable cost for the buyer, who in turn determines her retail price. When the supplier increases or decreases his wholesale price, the buyer reacts by changing the retail price accordingly.

When the two members make independent decisions, there are three possible scenarios of the price-setting game (Choi 1991): Nash, Supplier-Stackelberg, and Buyer-Stackelberg. It is obvious that a Stackelberg leader can achieve a higher profit than that from a Nash game. On the other hand, the total system's combined profit of the system may increase or decrease depending on the characteristics of the underlying demand function (Lee and Staelin 1997).

However, when the supply chain members coordinate their prices so as to maximize the joint profit, the total payoff would be no less than those from the non-cooperative, independent-firms scenarios ($\Pi_J^* \geq \Pi_s^* + \Pi_b^*$, where Π_J, Π_s, Π_b respectively denotes the joint, the supplier's, and the retailer's profits). The price coordination can be achieved in various ways, including vertical integration, binding contract, rational conjecture, and incentive-compatible quantity discount (Jeuland and Shugan 1983, 1988). Among them, the incentive-compatible pricing scheme is arguably the most stable, since the wholesale price can be designed such that the buyer's myopic optimal retail price also maximizes the joint profit.

In an incentive-compatible quantity discount, an upstream member of the distribution channel (e.g., a supplier) can provide a downstream member (e.g., a buyer) a price-based incentive so that it is the best interest to the buyer to set the retail price at the joint-optimal level. The simplest wholesale price to achieve this goal is to set the wholesale price at the supplier's marginal cost. At this wholesale price, it is easy to see that the retail price that serves the buyer's self-interest is also the system-optimal price. Since the supplier does not have any margin at this wholesale price, however, he has to levy the desired profit through a fixed charge for the planning period (i.e., franchise fee). This is in fact a two-part pricing, which can be thought of as a quantity discount. If the buyer buys less than the joint optimal level for the planning period, he pays a higher average wholesale price.

Jeuland and Shugan (1983) develop a formal model of quantity discount based on an incentive compatible profit design that eliminates double margination between the two members. McGuire and Staelin (1986) derive a modified solution in the form of a two-part tariff that is simpler to implement. Moorthy (1987) shows that any pricing schedule, including a quantity surcharge, that can equate the buyer's marginal cost with his marginal revenue at the channel-optimal demand level will coordinate the channel, as long as the marginal cost is strictly below his marginal revenue (i.e., the marginal cost curve "cuts" the marginal revenue curve from below at the optimal demand level).

Transaction Efficiency Models

Working independently, Lal and Staelin (1984), Monahan (1984), and Chakravarty (1984) have shown that quantity discount can be used to coordinate economic order quantity (EOQ) in order to minimize the system's total operating cost. The total operating cost includes inventory holding, order processing, and facility set-up costs that are associated with the quantity the buyer orders each time.

The focus of the transaction efficiency approach is to offer an incentive to the buyer so that it is the buyer's best interest to use the joint-optimal order quantity, instead of his own EOQ, which minimizes only his own operating cost. Given the operating costs for the supplier and the buyer, the joint EOQ can be determined in a straightforward fashion. It can be shown that the former order quantity is greater than the latter one. The buyer's operating cost (order cost and inventory holding cost; see Weng 1995 for a detailed discussion of the cost factors) will increase when his order quantity deviates from his myopic EOQ. Thus, in order to induce the buyer to order a larger quantity, a quantity discount can be used. An incentive-compatible quantity discount needs to be designed such that the buyer's total cost, including the price of the product, is minimized at that joint EOQ. By definition, the combined operating cost at the joint EOQ is less than that at the myopic EOQ. The difference is the efficiency gain for the supply chain (Dada and Srikanth 1987).

The simplest quantity discount schedule is to charge the original unit price if the order is less than the joint EOQ, and charge a discounted unit price when the order exceeds that (i.e., all-unit discount). There is a range of discounted wholesale price that can be applied for the purpose. The minimum necessary discount can set to the level that just compensates the buyer's increased cost due to the increased order quantity. In this case, the supplier takes the all efficiency gain. The maximum possible discount can be found by equating the supplier's total cost saving due to the increased order size with the lost sales revenue due to the price cut. In this case, the buyer takes all the efficiency gain. Also, several incremental quantity discounts have been proposed as variations of this all-unit discount (Dada and Srikanth 1997). The supplier can set the discounted wholesale price within this range based on the desired split of the efficiency gain between the two members. Kohli and Park (1989) propose several methods of dividing the total gain using bargaining mechanisms.

Combined Efficiency Models

In the above discussion, we have seen two separate sources of total channel gain when the channel members coordinate their actions. In both cases, by providing a wholesale price quantity discount, the supplier can induce the buyer to choose a decision (order quantity or retail margin) that is optimal for the whole channel. However, there are some differences in these two forms of quantity discounts. In the transaction efficiency models, the quantity discount is based on the *quantity per order*. The yearly demand is assumed fixed, and the retail price is irrelevant. The discount types are all-unit and incremental quantity discount schedules, which do not have the fixed charge component. In the elastic demand model, on the other hand, the discount is based on the *yearly (or per planning horizon) demand*. The simplest discount schedule is the two-part tariff, in which the marginal wholesale price equals the supplier's cost and a periodic fixed fee is charged in order to meet the supplier's profit goal.

When we combine the above two sources of gain, it is intuitive that the solution would combine an all-unit or incremental discount with a periodic fixed fee. Weng (1995) shows that this combined quantity discount is indeed sufficient for channel coordination. Thus, when transaction efficiency and elastic demand models are combined, both kinds of quantity discount schedules are required: (a) an all-unit or incremental quantity discount to motivate the buyer to choose the joint EOQ, and (b) a two-part pricing which further reduces the unit wholesale price to the supplier's marginal cost. Note that because the supplier recovers his profit by charging a periodic fixed fee, the average wholesale price becomes greater than the marginal wholesale price. However, Chakravarti and Martin (1989) show that the elastic demand function makes it extremely difficult to derive an analytical solution. Thus, these combined efficiency models resort to numerical procedures to determine quantity discount schedules. In particular, Chakravarti and Martin (1989) show that the optimal discounted price in their flexible demand problem is a singular value, instead of a range.

To our knowledge, all supply chain coordination studies to date in both marketing and operations management deal with two members (a supplier and a buyer). In reality, however, many supply chain problems consist of more than two participants, and it is important to extend the methodology to include other types of members. Also, the supplier's decision can be extended to include other variables than wholesale price. In the following, we present a model of supply chain coordination with three participants—a supplier, a buyer, and a logistics partner, and include a quality decision as part of the demand system. As in other studies, we use a numerical approach due to the functional complexity of elastic demand.

3. Three-Partner Model: Channel Coordination Involving Logistics Partners

Our analysis is based on a supply chain process with a *supplier* (a manufacturer or a wholesaler), a *buyer* (a retailer or a distributor who directly faces the market demand), and a third party logistics partner or a *transporter* who transports the shipment from the supplier to the buyer. We assume that the operating costs, including set up costs per order and the holding costs (per unit per year) incurred to all the partners, are known. In addition, we assume the market demand to the product is deterministic, continuous, decreases as buyer's (retailer's) market selling price x increases, and increases as supplier's product quality q improves (or as the supplier's investment on quality improvement increases). Note that in many functional product markets such as copying papers, cereals, and bottle waters, a basic quality level must be assured for the sales or FDA approval. Since these functional products are usually highly price-sensitive (Fisher, 1997) and price-competitive, any marginal price reduction, together with a quality enhancement, have the potential to further improve the market share.

Notations used in our analysis are summarized as follows:

q: The product quality level ($0<q<1$), where $q \Rightarrow 1$ indicates the highest quality attainable;

c: The supplier's variable cost of manufacturing($c<p$);

u: The supplier's budget on investing in the quality improvement for each unit of product, $u = \varphi(q)$;

$T(p)$: The transporter's unit shipping (including insurance) cost, as an increasing function of p;

g: Shipping rate charged by the transporter, $g>T(p)$;

k: The transporter's profit per unit of shipment, as a constant component of g;

S_t: The transporter's fixed cost per order;

H_t: The transporter's unit holding cost for inventory-in-transit per year;

$D(x,q)$: The annual market demand, as a function of x and q.

We assume that the supply process is a free-on-board (FOB)-destination process, which requires the supplier to pay for the shipping cost. This implies that the supplier's selling price p includes a variable production cost c, a unit product quality cost u (as one of the decision variables of the model), the shipping rate g, and the profit that the supplier may want to charge from his/her sales to the buyer, or $c + u + g < p < x,$ where x stands for the buyer (or retailer)'s market selling price. Given the assumptions, the yearly profitability of the supplier, the transporter and the buyer can be represented as

Supplier: $$\Pi_s(p,q) = (p - c - u - g)D(x,q) - S_s\, D(x,q)/Q - H_s\, Q/2 \tag{1}$$

Transporter: $$\Pi_t(g) = (g - T(p))D(x,q) - S_t\, D(x,q)/Q - H_t\, Q/2 \tag{2}$$

Buyer: $$\Pi_b(x) = (x - p)D(x,q) - S_b\, D(x,q)/Q - H_b\, Q/2 \tag{3}$$

The transporter's cost function contains following three components. First, for each unit shipped by the transporter, there is a variable cost, $T(p)$, which represents fuel, labor, and insurance for spoilage and damage that the transporter pays. This variable cost is a function of the unit selling price p due to the fact that liability and value of the goods have been an important factor in determining the shipping rate charged by carriers, according to National Motor Freight Classification 100-P (see Coyle, Bardi and Langley, 2003, for details). This leads to our first cost component, $T(p) \cdot D(x,q).$ Second, a major trend in today's logistics practice is carrier's responsibility for managing supplier's distribution centers (DCs). Examples of this include Penske for Whirpool (Penske News Release, 1998) and Ryder for Xerox (Konezny and Beskow, 1999). This implies a transporter's cost on handling the inventories at DCs, which in turn leads to our second cost component, $H_t \cdot Q/2,$ where Q stands for the buyer's order quantity. Finally, there always a fixed overhead cost for each order and shipment handled by the transporter, which covers the expenses for crew, truck maintenance, and terminal facility use, etc. We use S_t to denote this fixed cost per order and $S_t \cdot D(x,q)/Q$ for the transporter's annual fixed cost component.

As we can see, these three profit functions are optimized by different ordering quantities ($Q_s = \sqrt{2S_s\, D(x,q)/H_s}$, $Q_t = \sqrt{2S_t\, D(x,q)/H_t}$ and $Q_b = \sqrt{2S_b\, D(x,q)/H_b}$). As we can also see, the operation cost of the transporter, defined as a component of (2), is a *concave* function of the market demand.

3.1. Models for individual partner's yearly profit

When each partner acts for his/her own interest without coordination, it is common in practice to have the following business environment:

Definition 1. An *independent business environment* refers to a decentralized business process where the transporter decides on the shipping rate *g*. The buyer makes decisions on ordering quantity, or Q_b, that minimizes his/her own yearly fixed ordering and inventory holding cost. The supplier determines selling price p and quality level q with a reference to the market benchmarks (Cook and Jackson 2001) that together maximize his/her yearly profit. With any given p and q, the buyer then determines market selling price x that maximizes buyer's yearly profit.

In such a business environment, each partner seeks to optimize his/her own profit. In the absence of coordination, the buyer chooses the order quantity that maximizes his own profit, which is conditional on the demand. On the other hand, the demand is a function of the retail price as well as the quality level of the product, and the retail price depends on the wholesale price of the supplier. It is straightforward to see that the buyer's myopic EOQ, $Q_b = \sqrt{2S_b\, D(x,q)/H_b}$, is uniquely determined at any given retail price x . Therefore, with any given p and q, the buyer's decision is to determine the retail price $x = x_b^*(p,q)$ which maximizes his yearly profit:

$$\Pi_b(x \mid Q_b) = (x - p)D(x,q) - [2S_b H_b D(x,q)]^{1/2}. \tag{3a}$$

Given the buyer's optimal decision $x_b^*(p,q)$ and $Q_b(p,q)$, the transporter's yearly profit becomes

$$\Pi_t(g) = (g - T(p))D(x_b^*(p,q),q) - (S_t/S_b + H_t/H_b)[S_b H_b\, D(x_b^*(p,q),q)/2]^{1/2} \tag{2a}$$

and the supplier's yearly profit becomes

$$\Pi_s(p,q) = (p - c - u - g)D(x_b^*(p,q),q) - (\frac{S_s}{S_b} + \frac{H_s}{H_b})[S_b H_b\, D(x_b^*(p,q),q)/2]^{1/2} \tag{1a}$$

Since the supplier has a complete information about the buyer's Q_b and $x_b^*(p,q)$, he can always optimize the values of p and q to maximize Π_s upon a given demand function $D(x,q)$.

The combined supply chain profit under such an independent (decentralized) business environment can now be represented as (see Choi, Lei and Wang (2004), for details)

$$\begin{aligned}\Pi_s + \Pi_t + \Pi_b &= (x_b^*(p,q) - c - u - T(p))D(x_b^*(p,q),q) \\ &\quad - 2[1 + (\frac{S_s + S_t}{S_b} + \frac{H_s + H_t}{H_b})/2][S_b H_b\, D(x_b^*(p,q),q)/2]^{1/2}\end{aligned} \tag{4}$$

Note that (4) remains the same regardless whether the contract between the supplier and the buyer is based on a FOB-destination agreement (the supplier's expense on shipping cost) or a FOB-origin agreement (the buyer's expense on shipping cost) because it is only a transfer price between the two players.

3.2. Models for channel coordination

We now investigate the potential improvement on supply chain profitability by a centralized coordination or a *total supply chain coordination* where all the three partners are willing to make joint decisions to maximize (and then share) the total profitability of the supply chain as a whole entity.

In this environment, the joint yearly profit of the supplier, the buyer, and the transporter becomes

$$\Pi_J(x,p,q,Q) = \Pi_s + \Pi_t + \Pi_b$$

$$= (x-c-u-T(p))D(x,q) - (S_s + S_b + S_t)\,D(x,q)/Q - (H_s + H_b + H_t)\,Q/2$$

$$= (x-c-u-T(p))D(x,q) - S_J\,D(x,q)/Q - H_J\,Q/2 \tag{5}$$

where $S_J = S_s + S_b + S_t$, $H_J = H_s + H_b + H_t$.

The optimal ordering quantity that minimizes the joint operation costs (holding plus ordering) of all the three partners is

$$Q_J = \sqrt{\frac{2S_J D(x,q)}{H_J}} \tag{6}$$

which maximizes the total supply chain profitability (5) at

$$\Pi_J(x,p,q\,|\,Q_J) = (x-c-u-T(p))D(x,q) - [2S_J H_J D(x,q)]^{1/2} \tag{5a}$$

for any given supplier's *p, q*, transporter's *g*, and buyer's price *x*.

Theorem 1. (Dominance relationship) For any given *p*, *q*, *g* and *x*, the total supply chain profitability under Q_J dominates the *sum* of individual partners' maximum yearly profit in an independent business environment using ordering quantity Q_b. That is

$$\Pi_J(x,p,q\,|\,Q_J) \geq \Pi_s + \Pi_t + \Pi_b.$$

Proof: From (1a), (2a) and (3a), we have

$$
\begin{aligned}
&\Pi_s + \Pi_t + \Pi_b \\
&= (x - c - \varphi(q) - T(p))D(x,q) \\
&\qquad\qquad -2[1 + (\frac{S_s + S_t}{S_b} + \frac{H_s + H_t}{H_b})/2][S_b H_b D(x,q)/2]^{1/2}
\end{aligned}
$$

Since

$$
\begin{aligned}
&\Pi_J(x, p, q \mid Q_J) \\
&= (x - c - \varphi(q) - T(p))D(x,q) \\
&\quad - 2[1 + \frac{S_s + S_t}{S_b} + \frac{(S_s + S_t)(H_s + H_t)}{S_b H_b} + \frac{H_s + H_t}{H_b}]^{1/2}[S_b H_b D(x,q)/2]^{1/2}
\end{aligned}
$$

and

$$
[1 + (\frac{S_s + S_t}{S_b} + \frac{H_s + H_t}{H_b})/2]^2 - [1 + \frac{S_s + S_t}{S_b} + \frac{(S_s + S_t)(H_s + H_t)}{S_b H_b} + \frac{H_s + H_t}{H_b}] \geq 0,
$$

the claim holds. ■

In a very similar way, we can prove the following result.

Theorem 2: (Generalized dominance relationship) For any given p, q, g and x, we have

$\Pi_J(x, p, q \mid Q_J) \geq \Pi_s + \Pi_t + \Pi_b$

regardless the ordering quantity Q for the independent business environment is decided by which partner, or say regardless $Q = Q_s$, Q_b, *or* Q_t.

Theorem 3: The total supply chain profitability achieved under joint ordering quantity Q_J, $\Pi_J^*(p)$, increases as the supplier's unit selling price p decreases.

Proof: Assume that $p_1 < p_2$, then

$$\Pi_J^*(p_1) = \Pi_J^*(x_J^*, q_J^* \mid p_1)$$
$$= (x_J^*(p_1) - c - u_J^*(p_1) - T(p_1))D(x_J^*(p_1), q_J^*(p_1)) - [2S_J H_J D(x_J^*(p_1), q_J^*(p_1))]^{1/2}$$

$$\Pi_J^*(p_2) = \Pi_J^*(x_J^*, q_J^* \mid p_2)$$
$$= (x_J^*(p_2) - c - u_J^*(p_2) - T(p_2))D(x_J^*(p_2), q_J^*(p_2)) - [2S_J H_J D(x_J^*(p_2), q_J^*(p_2))]^{1/2}$$

$$\begin{aligned}\Pi_J^*(p_1) &\geq (x_J^*(p_2) - c - u_J^*(p_2) - T(p_1))D(x_J^*(p_2), q_J^*(p_2)) - [2S_J H_J D(x_J^*(p_2), q_J^*(p_2))]^{1/2} \\ &> (x_J^*(p_2) - c - u_J^*(p_2) - T(p_2))D(x_J^*(p_2), q_J^*(p_2)) - [2S_J H_J D(x_J^*(p_2), q_J^*(p_2))]^{1/2} \\ &= \Pi_J^*(p_2)\end{aligned}$$

■

Theorem 3 reveals the importance of supplier's coordination. If the supplier is willing not to charge a profit from his/her sales to the buyer, then the total supply chain profitability will be higher than otherwise they may achieve. This observation also leads to the following result.

Corollary 1: In a joint channel coordination environment, for any given *g*, the supplier's optimal unit selling price p_J^* are given by $\Pi_s(p_J^*) = 0$ or

$$p_J^* = c + u_J^* + g + S_s / Q_J + H_s Q_J /[2D(x_J^*, q_J^*)]$$

In addition, we have

Theorem 4: The total supply chain profitability achieved under joint ordering quantity Q_J, $\Pi_J^*(g)$, increases as the transporter's shipping rate *g* decreases.

Proof: Assume that $g_1 < g_2$, then

$$\Pi_J^*(g_1) = \Pi_J^*(x_J^*, q_J^*, p_J^* \mid g_1)$$
$$= \{x_J^*(g_1) - c - u_J^*(g_1) - T[c + u_J^*(g_1) + g_1 + (S_s/S_J + H_s/H_J)(S_J H_J D(x_J^*(g_1), q_J^*(g_1))/2)^{1/2}]\}$$
$$D(x_J^*(g_1), q_J^*(g_1)) - [2S_J H_J D(x_J^*(g_1), q_J^*(g_1))]^{1/2}$$

$$\Pi_J^*(g_2) = \Pi_J^*(x_J^*, q_J^*, p_J^* \mid g_2)$$
$$= \{x_J^*(g_2) - c - u_J^*(g_2) - T[c + u_J^*(g_2) + g_2 + (S_s/S_J + H_s/H_J)(S_J H_J D(x_J^*(g_2), q_J^*(g_2))/2)^{1/2}]\}$$
$$D(x_J^*(g_2), q_J^*(g_2)) - [2S_J H_J D(x_J^*(g_2), q_J^*(g_2))]^{1/2}$$

$$\Pi_J^*(g_1) \geq \{x_J^*(g_2) - c - u_J^*(g_2) - T[c + u_J^*(g_2) + g_1 + (S_s/S_J + H_s/H_J)(S_J H_J D(x_J^*(g_2), q_J^*(g_2))/2)^{1/2}]\}$$
$$D(x_J^*(g_2), q_J^*(g_2)) - [2S_J H_J D(x_J^*(g_2), q_J^*(g_2))]^{1/2} \quad \blacksquare$$
$$> \{x_J^*(g_2) - c - u_J^*(g_2) - T[c + u_J^*(g_2) + g_2 + (S_s/S_J + H_s/H_J)(S_J H_J D(x_J^*(g_2), q_J^*(g_2))/2)^{1/2}]\}$$
$$D(x_J^*(g_2), q_J^*(g_2)) - [2S_J H_J D(x_J^*(g_2), q_J^*(g_2))]^{1/2} = \Pi_J^*(g_2)$$

Theorem 4 reveals the importance of transporter's coordination. If the transporter is willing not to charge a profit from his/her contract with the supplier (in a FOB-destination agreement), then the resulting total supply chain profitability will be higher than otherwise they may achieve. This observation then leads to the following result.

Corollary 2: In a joint channel coordination environment, the transporter's optimal shipping rate charged to the supplier, g_J^* is given by $\Pi_t(g_J^*) = 0$ or

$g_J^* = T(p_J^*) + S_t / Q_J + H_t Q_J / [2D(x_J^*, q_J^*)]$, with $k_J^* = 0$, where k_J^* is the optimal profit rate that transporter charges in a joint channel coordination environment.

Example. Suppose the demand is represented by a convex, downward curve: $D(x,q) = dq/x^2$, and the transportation charge can be written as $g = k + ap + S_t/Q + H_t\, Q/[2D(x,q)]$. Further, let us suppose that the operation and cost parameters take the following values: S_b=\$200, S_t=\$600, S_s=\$4000, H_b=\$25, H_s=\$20, H_t=20, $d = 2 \times 10^{10}$, c=\$40, a=10%, and k=\$8.

In an independent business environment, we can derive the optimal prices, quality and EOQ as follows: $p_b^* = \$126$, $q_b^* = 0.874$, $x_b^* = \$252$ and $Q_b = 2100$. Then the profits for the respective supply chain partners are

$$\Pi_b^* = (x_b^* - p_b^*)D(x_b^*, q_b^*) - [2S_b H_b D(x_b^*, q_b^*)]^{1/2} = 34.694\times 10^6,$$
$$\Pi_t^* = (g - ap_b^*)D(x_b^*, q_b^*) - (S_t/S_b + H_t/H_b)[S_b H_b\ D(x_b^*, q_b^*)/2]^{1/2} = 2.206\times 10^6,$$

$$\Pi_s^* = (p_b^* - c - g(p_b^*, q_b^*) - u_b^*)D(x_b^*, q_b^*) - (S_s/S_b + H_s/H_b)[S_b H_b\ D(x_b^*, q_b^*)/2]^{1/2}$$
$$= 15.289\times 10^6,$$

and $\Pi_s^* + \Pi_t^* + \Pi_b^* = \$52.189\times 10^6$.

In a total coordination environment, we can derive the optimal prices, quality and EOQ as follows: $p_J^* = \$52.7$, $q_J^* = 0.871$, $x_J^* = \$105.7$ and $Q_J = 15180$. Then we have

$$\begin{aligned}\Pi_J^* &= (x_J^* - c - u_J^* - ap_J^*)D(x_J^*, q_J^*) - [2S_J H_J D(x_J^*, q_J^*)]^{1/2}\\ &= \$81.943\times 10^6\\ &> \Pi_s^* + \Pi_t^* + \Pi_b^* = \$52.189\times 10^6.\end{aligned}$$

From this example, we can see that the total supply chain profitability achieved in a totally coordinated environment is 57% higher than in an independent business environment.

Note that the coordinated wholesale price p_J is much lower than the uncoordinated price p_b, and the coordinated order quantity Q_J is much greater than that of the uncoordinated order quantity Q_b, representing a quantity discount.

4. Conclusion

We extended the existing literature on incentive-compatible quantity discount for supply chain coordination to a model that includes a third partner—a logistics company. In addition, our model also expands the applicability of the discount mechanism by including the quality variable to be determined by the supplier. One remarkable feature of this mechanism is that it does not require the retailer to "cooperate" for the benefit of the whole channel. While cooperation may leave an incentive for a channel member to deviate from the prescribed action, an incentive-compatible coordination is a self-regulating mechanism.

In our quantity discount model, the manufacturer designs the wholesale pricing schedule to induce the retailer to set up the retail price as well as the order quantity to the levels that maximize the total channel profit. The channel efficiency stems from the increased market demand, while the transaction efficiency comes from minimizing the operating costs. When both efficiencies are combined, the model becomes substantially complicated, and closed-form solutions are difficult to derive. However, we were able to provide basic intuitions of the coordination mechanism using a numerical example. Our analysis shows that such a discount schedule can be practically designed once the demand and cost parameters are known.

On the other hand, our model is limited by the assumption of a deterministic demand function. In practice, the demand can be stochastic, and the inventory policy can be substantially complicated with various assumptions such as stock-out, back order, safety stock, and lead-time. Generalizing our basic model by including these features would be a fruitful future research topic. Also, we considered only a single retailer case. Extending the model into multiple retailers with different demand and cost structure would be a challenging but rewarding research topic that increases practicality of our current model.

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